We gratefully acknowledge the support of the

DFG Deutsche Forschungsgemeinschaft

ALGORITHMIC OPTIMIZATION

The Research Training Group on Algorithmic Optimization (ALOP)
presents the following Workshop

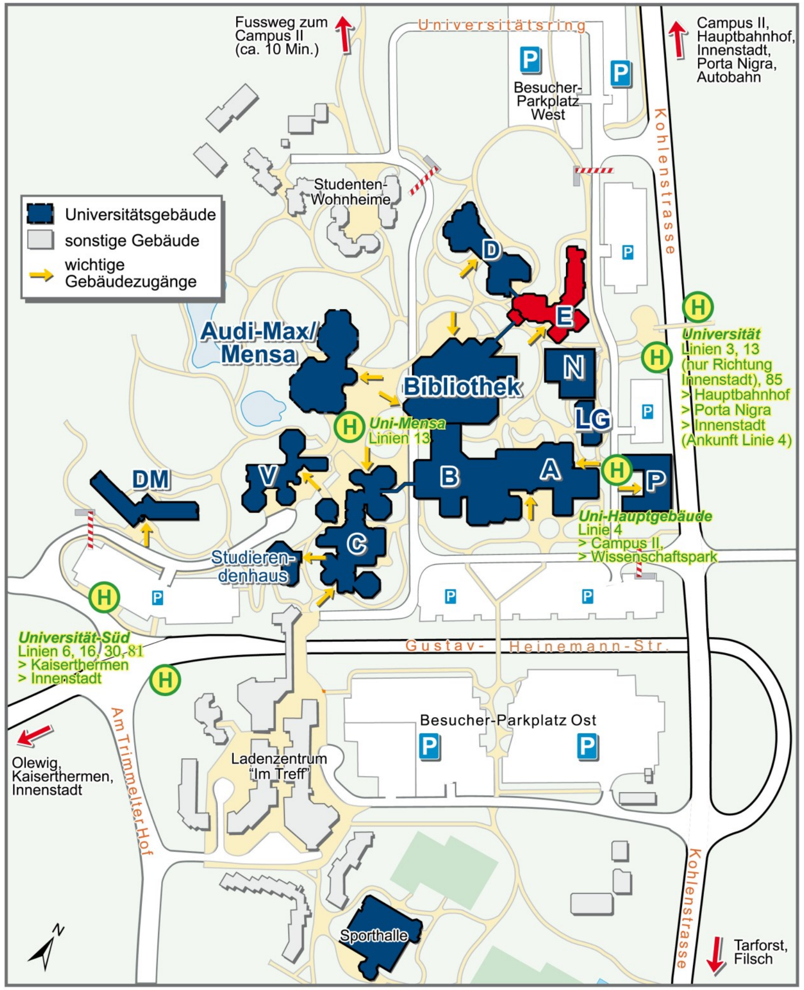
Reduced Order Models in Optimization

September 26—28, 2016
Trier University, Germany

Universität Trier

Location Map

The conference will take place in Building E (marked in red) on Campus I



Title: Model-Order Reduction in Computational Electromagnetics
Speaker: Romanus Dyczij-Edlinger
Affiliation: Lehrstuhl für Theoretische Elektrotechnik, Universität des Saarlandes

The talk covers selected techniques of model-order reduction (MOR) which are used in the numerical simulation of electromagnetic fields.

The finite-element discretization of linear, time-invariant (LTI) electromagnetic structures results in systems of linear equations and algebraic eigenvalue problems, respectively, that exhibit affine dependence on parameters such as frequency and material properties. A projection-based MOR framework that enables the efficient computation of the system response over a given parameter domain will be presented [1]. Moreover, the preservation of important system properties, including passivity, causality and reciprocity, by that method will be discussed [2].

Other numerical techniques, including the boundary-element method and the nearfield-to-farfield operator for computing antenna patterns, lead to integrals whose kernels depend non-affinely on operating frequency and geometry. The Empirical Interpolation Method (EIM) provides a powerful tool for constructing affine approximations. We will discuss efficient variants of the EIM and demonstrate their performance by considering a magneto-quasistatic problem involving rigid-body motion [3] and a broadband antenna array [4].

The material properties of ferromagnetic media are strongly field-dependent, and in electrical machines it is common to drive magnetic materials into saturation. To construct affine approximations, variants of the EIM are widely used. We propose an alternative approach, based on a third-order tensor, which results in a mathematical model that is directly accessible to projection-based MOR [5].

Finally, some preliminary results for optimizing the frequency response of LTI structures, using a quasi-Newton method, will be presented. We suggest constructing a reduced-order model (ROM) on-the-fly, in the course of the optimization [6]. First results indicate that the ROM, which is valid in the vicinity of the optimization trajectory only, is highly efficient in reducing the number of function evaluations of the underlying high-dimensional system [7].

References:

[1] S. Burgard, A. Sommer, O. Farle, R. Dyczij-Edlinger, "Reduced-Order Models of Finite-Element Systems Featuring Shape and Material Parameters," Electromagnetics, vol. 34, no. 3 – 4, pp. 143 – 160, 2014.

[2] R. Baltes, O. Farle, and R. Dyczij-Edlinger, "Time-Domain Simulation of Electromagnetic Fields Based on Reduced-Order Models Residing in the Frequency Domain," Proceedings of the IEEE International Conference on Electromagnetics in Advanced Applications (ICEAA 2015), pp. 1068 – 1071, Torino, Italy, Sep. 7 – 11, 2015.

[3] D. Klis, O. Farle, and R. Dyczij-Edlinger, "Model-order reduction for the finite-element boundary-element simulation of eddy current problems including rigid body motion," IEEE Trans. Magn., vol. 52, no. 3, March 2016, doi: 10.1109/TMAG.2015.2482541.

[4] A. Sommer, O. Farle, and R. Dyczij-Edlinger, "Certified Dual-Corrected Radiation Patterns of Phased Antenna Arrays by O_ine-Online Order Reduction of Finite-Element Models," J. Comput. Phys., vol. 299, pp. 22 | 44, 15 Oct. 2015.

[5] D. Klis, S. Burgard, O. Farle, and R. Dyczij-Edlinger, "A self-adaptive model-order reduction algorithm for nonlinear eddy current problems based on quadratic-linear modeling," IEEE Trans. Magn., vol. 52, no. 3, March 2016, doi: 10.1109/TMAG.2015.2487601.

[6] M. Mrozowski, "Complex 3D Filters and Multiplexer Design with Full-Wave Finite Element Code," IEEE-MTS International Conference on Numerical Electromagnetic and Multiphysics Modeling and Optimization (NEMO) for RF, Microwave and Terahertz Applications (NEMO 2016), Plenary Talk, Bejing, China, July 29 – 29, 2016.

[7] S. Brandl, "Effiziente mathematische Optimierung elektromagnetischer Strukturen mittels parametrischer Modellordnungsreduktion," Bachelorarbeit am Lehrstuhl für Theoretische Elektrotechnik, Universität des Saarlandes, Saarbrücken, Germany, Sep. 2016 (in German).

A HJB-POD approach to the control of the level set equation

A. Alla, G. Fabrini, M. Falcone

We consider an optimal control problem where the dynamics is given by the propagation of a one-dimensional graph controlled by its normal speed, which has been proposed in [3]. A target corresponding to the final configuration of the front is given and we want to minimize the cost to reach the target. We want to solve this optimal control problem via the dynamic programming approach but it is well known that this methods suffers of the "curse of dimensionality" so that we can not apply the method to the semi-discrete version of the dynamical system. However, this is made possible by a reduced-order model for the level set equation which is based on Proper Orthogonal Decomposition (see [4]). This results in a new low-dimensional dynamical system which is sufficient to track the dynamics. By the numerical solution of the Hamilton-Jacobi-Bellman equation related to the POD approximation we can compute the feedback law and the corresponding optimal trajectory for the nonlinear front propagation problem (see [1] and the reference therin for the HJB-POD approach). In particular for solving the Bellman equation we will use the accelerated policy iteration algorithm proposed in [2]. We will present some numerical tests to show the efficiency of the proposed method.

References

[1] A. Alla and M. Falcone. *An adaptive POD approximation method for the control of advection-diffusion equations*, in K. Kunisch, K. Bredies, C. Clason, G. von Winckel, (eds) *Control and Optimization with PDE Constraints*, International Series of Numerical Mathematics, **164**, Birkhäuser, Basel, 2013, 1-17.

[2] A. Alla, M. Falcone and D. Kalise. *An Efficient Policy Iteration Algorithm for Dynamic Programming Equations* SIAM J. Sci. Comput. **37** (2015), no. 1, 181-200.

[3] K. Deckelnick, C.M. Elliott, V. Styles. *Optimal control of the propagation of a graph in inhomogeneous media* SIAM J. Control. Optim. **48** (2009), no. 3, 1335-1352.

[4] S. Volkwein. *Model Reduction using Proper Orthogonal Decomposition*, Lecture Notes, University of Konstanz, 2011.

Reduced Order Models in Optimization
(ROM)

Reduced order models represent a growing research direction in the modeling, simulation and optimization of complex systems. During this workshop, taking place as part of the Research Training Group „Algorithmic Optimization," internationally known researchers will present their latest research results with a focus on particular aspects in the area of optimization.

September 26—28, 2016
Building E,
Trier University
Universitätsring 15
54296 Trier
Germany

Organizers:
Ekkehard Sachs, Volker Schulz, Martin Siebenborn, Stefan Volkwein

Program: **Monday, September 26, 2016**

10:00 am – 10:45 am	**John Burns**	Approximation for Design and Control of Composite Thermal Fluid Systems: Reduced Models
10:45 am – 11:30 am	**Karen Veroy-Grepl**	Certified Reduced Basis Methods in Optimal Control and Data Assimilation
11:30 am – 1:00 pm		*Lunch Break*
1:00 pm – 1:45 pm	**Bernd Noack**	Closed-loop turbulence control using reduced-order modeling and machine learning
1:45 pm – 2:30 pm	**Andrea Ferrero**	Different approaches to the development of reduced-order models for NS equations
2:30 pm – 3:00 pm		*Coffee Break*
3:00 pm – 3:45 pm	**Karl Meerbergen**	Progress in model reduction for systems with nonlinear frequency dependency
3:45 pm – 4:30 pm	**Laura Iapichino**	Reduced Basis methods for PDE constrained Multi-objective Optimization
5:00 pm – 8:00 pm		*Dinner Collaboration*

MOR in the optimization of highly nonlinear oscillatory electronic circuits

Schilders Wil
G.De Luca
TU Eindhoven,
Centre for Analysis, Scientific Computing and Applications, The Netherlands

The electronics industry has always been one of the most stimulating environments for techniques in the area of model order reduction. Starting with the advent of AWE, and the subsequent development of PVL, PRIMA and other methods, one always encounters examples from the electronics industry, more specifically for device and circuit simulation. Although slowly but steadily other application domains are entering the field, the electronics industry remains a prime source for challenges in model order reduction. Especially for nonlinear and highly oscillatory circuits, there is a big challenge, as simulation times are often a few days or even weeks, and this is not acceptable for designers. What's more, when looking at the solutions generated in the time domain, one gets the feeling that it should be possible to model the behavior in a more concise and compact way, as solutions only change very slowly from one period to another. However, how to capture this slowly changing behavior is a challenge in itself. As designers also seek to optimize such circuits, it is extremely important to significantly reduce computation times.

An important case of so- called quasi-periodic circuits with very long simulation time are phase locked loops (PLLs). PLLs find their applications in wireless systems, digital circuits and medical devices. Time domain analyses are performed by numerically integrating the circuits DAEs. For PLLs, the time step during integration is related to the PLLs output frequency f_{out}, which is often in the GHz range, meaning that a huge number of steps will be made. Designers simulate PLLs for a large period of time to extract important characteristics such as power consumption, locking time, phase noise and jitter. Previous work on speeding up the simulation of PLLs is based on partial or full replacement of PLL blocks by macromodels, others use full TL simulation (e.g., see [1, 2, 3]). However, none of them estimates all the PLL characteristics very well. In fact, promising initial work was done in the 1990s, but necessary improvements to truly speed up simulations were not found. We propose a technique to accurately extract all the aforementioned characteristics of PLLs while very substantially accelerating noise- free and noisy simulations, by replacing only the VCO and divider blocks in the PLL with a single phase macromodel, built through steady-state (SST) analyses [4, 5] and SST noise methods [6, 7] The method leads to a reduced order model for the full PLL. The methodology is applicable to other types of highly nonlinear and osciallatory devices, and we will also indicate future work on such devices that are even more challenging. The methodology could also be applied to other fields of application, where high frequent components are present.

References

[1] P. Maffezzoni and S. Levantino, Analysis of VCO phase noise in charge-pump phase- locked loops. *IEEE Trans. Circuits Syst. I: Regular Papers* , vol. 59, n. 10, pp. 2165- 2175, 2012.

[2] X. Lai and J. Roychowdhury, TP-PPV: piecewise nonlinear, time-shifted oscillator macromodel extraction for fast, accurate PLL simulation. *ICCAD*, Hassoun, Soha, 2006.

[3] K. Bittner and H. G. Brachtendorf, Adaptive multi-rate wavelet method for circuit simulation. *Radioengineering*, vol. 23, pp. 300-307, 2014.

[4] K. S. Kundert and A. Sangiovanni-Vincentelli, Finding the steady-state response of analog and microwave circuits. *Proceedings of the IEEE on Custom Integrated Circuits Conference*, pp. 6.1/1-6.1/7, 1988.

[5] R. Telichevesky, K. S. Kundert and J. K. White, Efficient Steady-State Analysis Based on Matrix-Free Krylov-Subspace Methods. *32nd Design Automation Conference (DAC)*, pp. 480-484, 1995.

[6] E. Ngoya, J. Rousset and D. Argollo, Rigorous RF andmicrowave oscillator phase noise calculation by envelopetransient technique. *IEEE MTT-S International Microwave Symposium Digest.*, vol. 1, pp. 91-94, 2000.

[7] A. Demir, A. Mehrotra and J. Roychowdhury, Phase noise in oscillators: a unifying theory and numericalmethods for characterization. *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 655-674, 2000.

Nonlinear robust optimization using a second order approximation technique and model order reduction

By Oliver Lass

We investigate a nonlinear constrained optimization problem with uncertain parameters. It is addressed by a robust worst-case formulation. The resulting optimization problem is of bi-level structure. This type of problems are difficult to treat computationally. We propose and investigate an approximate robust formulation that employs a quadratic approximation. Additionally we will mix the proposed framework with a linear approximation when appropriate.

The developed method is then applied to the optimal placement of a permanent magnet in the rotor of a synchronous machine. The goal is to optimize the volume and position of the permanent magnet while maintaining a given performance level. These quantities are computed from the magnetic vector potentials obtained by the magnetostatic approximation of Maxwell's equation with transient movement of the rotor. Permanent magnet synchronous machines can be described sufficiently accurate by this model. Hence we arrive at an optimization problem governed by a set of elliptic equations, where one partial differential equation has to be solved for every rotor position. This model formulation will give rise to the development of model order reduction. Due to manufacturing, there are uncertainties in material and production precision. Here the introduced robust optimization framework comes into play and accounts for uncertain model and optimization parameters.

The problem formulation as well as the robustification of the optimization lead to high computational cost that requires to investigate methods for efficient realization. Since the transient movement of the rotor can be interpreted as a multi query operation, model order reduction is a promising choice. By generating reliable reduced order models with a posteriori error control the computation can be accelerated. Numerical results are presented to validate the presented approach.

Program: Tuesday, September 27, 2016

9:00 am – 9:45 am	**Jeff Borggaard**	POD-Based ROMs of Fluids for Shape Optimization Problems
9:45 am – 10:30 am	**Jens Bremer**	Reduced Order Modeling and Optimization
10:30 am – 11:00 pm		*Coffee Break*
11:00 am – 11:45 am	**Martin Stoll**	Low-rank solvers for PDE-constrained optimization and statistical inverse problems
11:45 am – 12:30 pm	**Babak Maboudi**	Structure-Preserving Model-Reduction of Hamiltonian Systems
12:30 am – 2:00 pm		*Lunch Break*
2:00 pm – 2:45 pm	**Bernard Haasdonk**	RB Approximation for Parameter Optimization and Optimal Feedback Control
2:45 pm – 3:30 pm	**Martin Grepl**	A certified trust-region reduced basis approach to PDE-constrained optimization
3:30 pm – 4:00 pm		*Coffee Break*
4:00 pm – 4:45 pm	**John Singler**	POD computations and POD projections
4:45 pm – 5:30 pm	**Oliver Lass**	Nonlinear robust optimization using a second order approximation technique and model order reduction
7:30 pm – 10:00 pm		*Social Gathering “Das Weinhaus”*

Program: Wednesday, September 28, 2016

9:00 am – 9:45 am	**Wil Schilders**	MOR in the optimization of highly nonlinear oscillatory electronic circuits
9:45 am – 10:30 am	**Giulia Fabrini**	A HJB-POD approach to the control of the level set equation
10:30 am – 11:00 pm	*Coffee Break*	
11:00 am – 11:45 am	**Romanus Dyczij-Edlinger**	Model-Order Reduction in Computational Electromagnetics

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POD computations and POD projections

First, we discuss an incremental algorithm for proper orthogonal decomposition (POD) computations. We modify an incremental matrix SVD algorithm of Brand to accommodate data arising from Galerkin-type simulation methods for time dependent PDEs. The algorithm initializes and efficiently updates the POD eigenvalues and modes during the time stepping in a PDE solver without storing the simulation data. We demonstrate the effectiveness of the algorithm using finite element computations for fluid flows.

Next, we discuss properties of POD projections for data arising from time dependent PDEs. Specifically, we consider time varying data taking values in two different Hilbert spaces, and examine pointwise convergence and uniform boundedness of four different POD projections. We discuss known results, open problems, and potential applications.

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A certified trust-region reduced basis approach to PDE-constrained optimization

Parameter optimization problems constrained by partial differential equations (PDEs) appear in many applications in science and engineering. The PDE usually describes the underlying system or component behavior, while the parameters serve to identify a particular configuration of the component --- such as boundary and initial conditions, material properties, and geometry. Especially in applications involving many parameters, solving these optimization problems may require a prohibitively large number of computationally expensive PDE solves if classical discretization techniques such as finite elements (FE) or finite volumes (FV) are used. One way to decrease the computational burden is the surrogate model approach, where the original high-dimensional FE or FV model is replaced by its reduced order approximation.

In this talk, the reduced basis (RB) model reduction method is used in conjunction with a trust region optimization framework to accelerate PDE-constrained parameter optimization. We extend existing a posteriori error bounds for RB approximations and derive a posteriori error bounds for the RB cost and its gradient for quadratic cost functionals (e.g., a typical output least squares formulation). We then incorporated these bounds into the trust region framework, which allows to guarantee convergence of the trust region reduced basis approach to the optimum of the high-fidelity model. The proposed method uses high-fidelity solves to update the RB model only if the approximation is no longer sufficiently accurate, reducing the number of full-fidelity solves required. We consider problems governed by elliptic and parabolic PDEs and present numerical results for a thermal fin model problem.

Joint work with E. Qian (MIT), K. Veroy (RWTH Aachen), and K. Willcox (MIT).

Approximation for Design and Control of Composite Thermal Fluid Systems: Reduced Models

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Abstract

In this paper we consider efficient approximations of composite infinite dimensional systems. These systems arise naturally when the problem is multidisciplinary (e.g., aero-elasticity, thermal-fluids) but also occur when actuator dynamics are included in the control design problem. We present examples to highlight some technical issues that occur when dealing with interconnected systems and then focus on a special class of composite systems that occur when actuator dynamics with delays are included as part of the model. Reduced approximations are used to generate finite dimensional models. Although these models can be further reduced by applying numerical model reduction, caution must be exercised when the model is to be used for control. Numerical examples are presented to illustrate the ideas and to highlight the issues.

Certified Reduced Basis Methods in Optimal Control and Data Assimilation

Speaker: K. Veroy

The reduced basis method is a certified model order reduction technique for the rapid and reliable solution of parametrised partial differential equations, and it is especially suited for the many-query, real-time, and slim computing contexts. This talk begins with a brief introduction into the basic elements of the RB method: approximation, a posteriori error estimation, offline-online computational decomposition, and the greedy algorithm. We then tackle some recent contributions to the field, focusing on problems in optimal control and data assimilation. In particular, we present a certified RB approach to four dimensional variational data assimilation (4D-VAR) [Le Dimet, 1981]. Several contributions have explored the use of reduced order models as surrogates in a 4D-VAR setting (see, for instance, [Stefanescu et al., 2015]). In this work, we consider the particular case in which the behaviour of the system is modelled by a parametrised parabolic PDE in which the initial condition and model parameters (e.g., material or geometric properties) are unknown. The resulting formulation is a two level optimisation problem in which the misfit is minimised over both the control and the parameter space. We then build upon recent results on RB methods for optimal control problems in order to derive a posteriori error estimates for RB approximations to solutions of the 4D-VAR problem. This permits the rapid and reliable solution of the combined data assimilation and parameter estimation problem. Numerical tests using a convection diffusion problem with unknown initial condition and Peclet number are used to show the viability of the proposed approach.

RB Approximation for Parameter Optimization and Optimal Feedback Control

Speaker: Bernard Haasdonk

Abstract:

Reduced Basis (RB) methods are powerful model reduction techniques for rapid approximation of large scale parametric problems. This enables their use in multi-query settings, for example in parameter optimization or parametric optimal control problems. In this presentation we report on some of our work in these application areas.

First, we present a method for rapid and certified parameter optimization of problems with PDE constraints [1,2]. For time-dependent problems we make use of an RB-formulation for a general class of evolution problems covering standard iterative time-stepping schemes. The reduced spaces for such problems are beneficially constructed by the POD-Greedy procedure, for which we recently provided theoretical foundation by convergence rate proofs. Extensions of this procedure involve parameter- and time-partitioning approaches. We will demonstrate how these ingredients can be used in iterative direct parameter optimization problems. In addition to approximate surrogate optimization results, we provide rigorous a-posteriori error bounds for solutions, outputs, sensitivities and optimal parameters.

Second, we address optimal feedback control problems, which can be solved by Algebraic Riccati Equations (AREs) [3, 4]. The ARE is a quadratic matrix valued equation with a variety of important applications in the field of dynamical systems. As the modelling of many complex technical phenomena is usually performed by means of partial differential equations, which after semi-discretization in space yield very large time dependent ODE systems, the question arises how the resulting large AREs can be solved efficiently. Although many very efficient solution algorithms exist, real-time applications or many-query scenarios in a parametric case can lead to infeasible calculation times. For that reason we present a certified RB method for obtaining approximate solutions to the ARE rapidly and with reliable error statements. As one ingredient, we formulate a new procedure called Low Rank Factor Greedy algorithm, which exploits a low rank structure in the solution matrices to build a suitable low dimensional basis. Experiments demonstrate the effectivity of the approach and the application in optimal linear quadratic regulator (LQR) feedback control.

References:

[1] Dihlmann, M., Haasdonk, B.: Certified PDE-constrained parameter optimization using reduced basis surrogate models for evolution problems, COAP, Computational Optimization and Applications, 2015, 60, 753-787.

[2] Dihlmann, M., Haasdonk, B.: Certified Nonlinear Parameter Optimization with Reduced Basis Surrogate Models, PAMM, Proc. Appl. Math. Mech., 2013, 13, 3-6.

[3] Schmidt, A., Haasdonk, B.: Reduced Basis Approximation of Large Scale Algebraic Riccati Equations, SimTech Preprint, University of Stuttgart, 2015 Submitted to COCV.

[4] Schmidt, A., Haasdonk, B.: Reduced basis method for H2 optimal feedback control problems, Proc. of CPDE 2016.

Structure-preserving model reduction of Hamiltonian systems
By Babak Maboudi

Reduced basis methods have emerged as a powerful approach for the reduction of the intrinsic complexity of parametrized partial differential equations. Despite the great success of these techniques over the last decade, the stability of the reduced systems over long time integration remains an open question. In the context of Hamiltonian and Lagrangian systems, recent works suggest a modification of proper orthogonal decomposition method (POD) to preserve the geometric structure, yielding a stable reduced system. In my talk, I present a greedy approach for construction of a reduced system that preserves the symplectic structure of Hamiltonian systems. This method, constructs a reduced Hamiltonian, as an approximation to the Hamiltonian of the original system and hence, guaranteeing stability over long time integration. In addition, I demonstrate how to use this technique to preserve divergence properties of an incompressible vector field in the reduced model.

Workshop 'Reduced Order Model' of the Graduate School on 'Mathematical Optimization'
University of Trier, Germany, 2016-09-26..28

Closed-loop turbulence control using reduced-order modeling and machine learning[1] [2]

Bernd Noack[3]
LIMSI-CNRS, France; PPRIME, France & TU Braunschweig, Germany

Active turbulence control is a rapidly evolving, interdisciplinary field of research. In particular, closed-loop control with sensor information can offer distinct benefits over blind open-loop forcing. The range of current and future engineering applications of closed-loop turbulence control has truly epic proportions, including cars, trains, airplanes, jet noise, air conditioning, medical applications, wind turbines, combustors, and energy systems. Many problems of the University of Trier belong to that portfolio.

A key feature, opportunity and technical challenge of closed-loop turbulence control is the inherent nonlinearity of the actuation response. For instance, excitation at a given frequency will affect also other frequencies. This frequency cross-talk is not accessible in any linear control framework. We propose a novel nonlinear control strategy with reduced-order modeling (ROM) for known actuation mechanisms (exploitation) and machine learning techniques for discovering unknown mechanisms (exploration). First, successful ROM-based studies for drag reduction of a bluff body and lift increase of an airfoil are reviewed. Then, machine learning control (MLC) is proposed. This model-free method has significantly outperformed existing control strategies in several shear-flow experiments, for instance the backward facing step at PMMH and UVHC, the TUCOROM mixing layer at P' or turbulent boundary layer separation at PRISME & LML.

BIO: Bernd NOACK develops closed-loop flow control solutions for cars, airplanes and transport systems — in an interdisciplinary effort with dedicated colleagues, PostDocs and PhD students at LIMSI, PPRIME and TU Braunschweig — in collaboration with the groups of Profs. M.W. Abel, S. Brunton, E. Kaiser, L. Keirsbulck, A. Kourta, S. Krajnović, M. Morzyński, R.K. Niven, C.O. Paschereit, B. Protas and industry. He is Director of Research CNRS at LIMSI-CNRS, France, visiting scholar at Institute PPRIME, France, and Professor at TU Braunschweig, Germany. He has co-authored over 200 publications, 2 patents and 2 Springer books on turbulence control. His work has been honored by numerous awards, e.g. a Fellowship of the American Physical Society.

Subset of the turbulence control team
at the TUCOROM wind-tunnel.
From top left to bottom right:
S. Brunton, V. Parezanovic, J.-C. Laurentie,
M. Segond, T. Duriez and B.R. Noack.

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[1] On invitation of Prof. Ekkehard Sachs.

[2] The talk comprises joint work with the teams of Markus Abel, Jean-Luc Aider, Steven Brunton, Thomas Duriez, Eurika Kaiser, Rudibert King, Laurent Keirsbulck, Azeddine Kourta, Marek Morzyński, Robert Niven, Michel Stanislas as well as the former Poitiers' TUCOROM Team (Jean-Paul Bonnet, Laurent Cordier, Vladimir Parezanovic, and Andreas Spohn).

[3] `http://BerndNoack.com`

Different approaches to the development of reduced-order models for NS equations

Michel Bergmann, Andrea Ferrero, Angelo Iollo

Institut de Mathématiques de Bordeaux, Université de Bordeaux and Memphis Team - Inria Bordeaux - Sud Ouest

In this contribution we explore some numerical alternatives to derive efficient and robust low-order models of the Navier–Stokes equations. The first considered approach is based on an hybrid CFD-ROM approach in which the ROM is used to define the boundary conditions of a CFD simulation. This makes possible to reduce significantly the size of the domain studied by the CFD solver. An alternative approach, based on residual minimization, is presented in the following. We start from the fact that classical Galerkin or Petrov-Galerkin approaches for ROM can be derived in the context of a residual minimization method similar to variational multi scale modelling , VMS [1]. Based on this, we introduce a residual minimization scheme that directly includes VMS stabilizing terms in the low-order model as proposed in [2], [3]. Here, however, the unknowns of the minimization problem are the union of the coefficients of a modal representation of the solution and of the physical unknowns at certain collocation points [4]. The modal representation is typically based on empirical eigenfunctions obtained bye proper-orthogonal decomposition, whereas the residual at collocation points are obtained by an adaptive discretization. Examples relative to model problems and moderately complex flows will be presented.

[1] Hugues TJR, Feijoo G, Mazzei L, Quincy JB. The variational multiscale method —a paradigm for computational mechanics. *Computer Methods in Applied Mechanics and Engineering* 1998; **166**:3–24.
[2] Bergmann M, Bruneau C, Iollo A. Enablers for robust pod models. *Journal of Computational Physics* 2009; **228**(2):516–538.
[3] Weller J., Lombardi E., Bergmann, Iollo A. Numerical methods for low-order modeling of fluid flows based on POD *Int. J. Numer. Meth. Fluids* 2009.
[4] Buffoni, M., Telib, H., & Iollo, A. (2009). Iterative methods for model reduction by domain decomposition. *Computers & Fluids*, 38(6), 1160–1167. http://doi.org/10.1016/j.compfluid.2008.11.008

Low-rank solvers for PDE-constrained optimization and statistical inverse problems

Martin Stoll, Max-Planck-Institut Magdeburg

Abstract:

Solving PDE-constrained optimization problems typically leads to large linear systems where we suffer from the curse of dimensionality regarding the discretization in space and time. This effect is even more pronounced when additional parameters or uncertainties are involved.

In order to break this curse, we introduce a low-rank in time technique that can be used with iterative solvers. To illustrate the potential of this technique, we additionally consider statistical inverse problems where we look at the problem of estimating the uncertainty in statistical inverse problems using Bayesian inference. When the probability density of the noise and the prior are Gaussian, the solution of such a statistical inverse problem is also Gaussian. Therefore, the underlying solution is characterized by the mean and covariance matrix of the posterior probability density. However, the covariance matrix of the posterior probability density is full and large, computation of such a matrix is impossible for a large dimensional parameter space.

It is known that for many ill-posed problems, the Hessian matrix of the data misfit part has low numerical rank and is therefore possible to perform a low-rank approach to approximate the posterior covariance matrix. For such low-rank approximation, one needs to solve a forward partial differential equation (PDE) in space and time domain and another adjoint PDE in both space and time domain. This in turn gives $\mathcal{O}(N_sN_t)$ complexity for both computations and storage, where N_s is the dimension of the space domain and N_t is the dimension of the time domain. Such computations and storage are not possible for large problems. To get over this obstacle, we develop a new approach that utilizes recently developed low-rank in time algorithm together with the low-rank Hessian method. We reduce both the computational complexity and storage requirement from $\mathcal{O}(N_sN_t)$ to $\mathcal{O}(N_s + N_t)$.

We show some first numerical experiments to illustrate the advantages of such approach.

Reduced Order Modeling and Optimization of CO2 Methanation Reactors (by Jens Bremer)

Utilizing volatile renewable energy sources (e.g., solar, wind) for chemical production systems requires a deeper understanding of their dynamic operation modes.

Taking the example of a methanation reactor in the context of power-to-gas applications, a dynamic optimization approach is used to identify control trajectories for a time optimal reactor start-up avoiding distinct hot spot formation. For the optimization, we develop a dynamic, two-dimensional model of a packed-bed tube reactor for carbon dioxide methanation which is based on the reaction scheme of the underlying exothermic Sabatier reaction mechanism. However, dealing with dynamic, nonlinear, large-scale process models often leads to many computational difficulties. Facing this issue, snapshot-based model order reduction techniques, such as proper orthogonal decomposition together with the discrete empirical interpolation method (POD-DEIM) generate considerably less complex models featuring a lower number of states. This surrogate models show a significant potential for dynamic optimization tasks in terms of less CPU and memory costs while guaranteeing an acceptable model error at the same time.

We will discuss the methodology and feasibility for optimal control of the methanation reactor and outline its further potential for integrating reduced order models.

Karl Meerbergen

Title:
Progress in model reduction for systems with nonlinear frequency dependency

Abstract:

(Joint work with Roel Van Beeumen, Wim Michiels and Pieter Lietaert)

Classical models in mechanical and civil engineering are second order finite element systems, usually represented by three matrices: mass, stiffness and damping. For new materials, the damping matrix is often frequency dependent. The dependency can be rather complex, but is usually nonlinear. In this talk, we show how techniques initially developed for nonlinear eigenvalue problems can be used for model order reduction of such problems. We present the Compact Rational Krylov method (CORK) as the computational work horse. Then, we illustrate how it can be used to build a linear reduced model of a nonlinear model in the Laplace or frequency variable. The idea is to reduce a large linear model obtained from interpolating the nonlinear model. Finally, we will also show how two-sided models can be built in a dynamic way.

Reduced Basis methods for PDE Constrained Multi-objective Optimization

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Problems where several objective functions have to be simultaneously optimized, often arise in many applicative contexts. Compared with optimal control problems with a single objective functions, multiobjective problems are more complex and require longer computational times to be solved. Reduced order techniques for the numerical solution of multiobjective optimization with linear and semilinear PDEs constraints are proposed. The aim is to find solutions in reasonable computational times which do not penalize the optimization of any objective function and which represent a good compromises for all the individual ones. In general, does not exist a single optimal solution, but there exists a (possibly infinite) number of optimal solutions, called Pareto points. In the multiobjective optimization theory, the Pareto optimality allows to determine efficient optimal points for all the considered objective functions [?]. We apply the reduced basis method [?] in this context to handle the computational complexity and resolution times of the problem and, at the same time, to ensure a suitable level of the solution accuracy.

References

[1] C. Hillermeier. Nonlinear multiobjective optimization. A generalized homotopy approach. Birkhaeuser Verlag, Basel, 2001

[2] J. S. Hesthaven, G. Rozza, B. Stamm. Certified Reduced Basis Methods for Parametrized Partial Differential Equations. Springer. 2016

POD-Based ROMs of Fluids for Shape Optimization Problems

In this talk, we discuss the incorporation of shape derivatives of Navier-Stokes simulations to build parametric reduced-order models for shape optimization problems. The methodology simultaneously computes POD modes as well as their parametric derivatives. These, combined with on-line extrapolation or off-line interpolation strategies, can be used as surrogate models in optimization applications. The algorithm will be presented along with accompanying numerical studies that include the Navier-Stokes equations as the PDE-constraint. We will also discuss some details of potential extrapolation and higher-order interpolation methods that could be implemented using these reduced-order models.

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