Network equilibrium for robust optimal rapid transit network design

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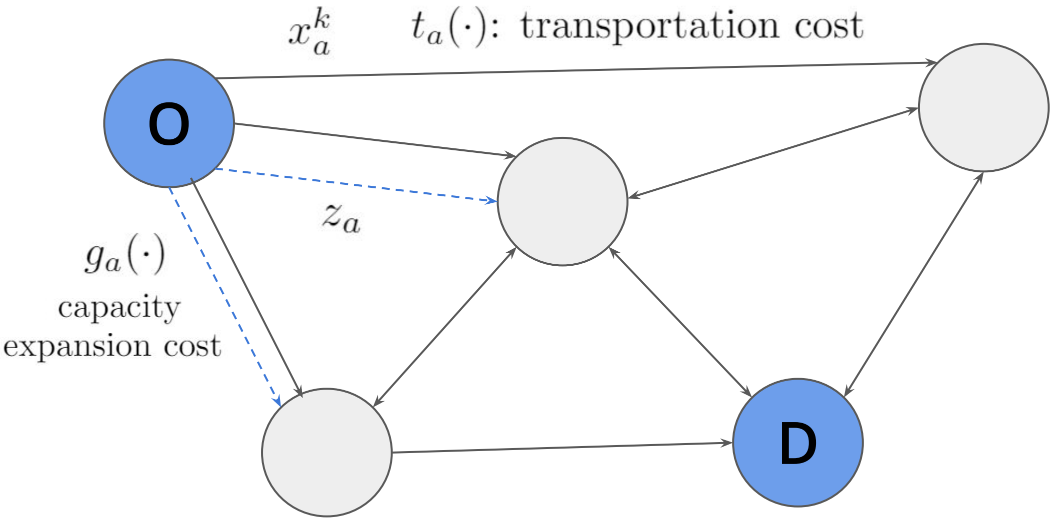
ALOP Autumn School on
Bilevel Optimization

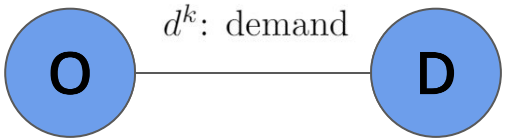
Bilevel Programming Problem

Network Design Problem with selfish users

Mathematical formulation



$G = (N, A)$



$\Omega \in N \times N$: set of OD pairs.

$x^k = (x_a^k)_{a \in A}$: flow vector

$x_a = \sum_{k \in \Omega} x_a^k$: total flow

Bilevel Programming Problem

Network Design Problem with selfish users

$$\min \sum_{a \in A} t_a(x_a, z_a) x_a$$

First Level: Minimize travel time costs and improve the network

$$\text{s.a} \sum_{a \in A} g_a(z_a) \leq B$$

$$z \geq 0$$

$$x \in argmin \left\{ \sum_{a \in A} \int_o^{v_a} t_a(y, z_a)\, dy : Nv^k = d^k \quad \forall k \in \Omega, \quad v \geq 0 \right\}$$

Second Level: Users behave according to Wardrop's first principle of traffic equilibrium

Reformulation of Bilevel Programming Problem

$$\min \sum_{a \in A} t_a(x_a, z_a) x_a$$

$$\text{s.a} \quad \sum_{a \in A} g_a(z_a) \leq B$$

$$\sum_{a \in A} t_a(x_a, z_a) x_a \leq \sum_{a \in A} t_a(x_a, z_a) v_a \quad \forall v \ \text{feasible}$$

$$N x^k = d^k \quad \forall k \in \Omega$$

$$x, z \in R^+$$

Variational inequality problem for traffic equilibrium

Reformulation of Bilevel Programming Problem

Bureau of Public Roads (BPR) function

$$\min \sum_{a \in A} t_a(x_a, z_a) x_a$$

$$\text{s.a} \quad \sum_{a \in A} g_a(z_a) \leq B$$

$$\sum_{a \in A} t_a(x_a, z_a) x_a \leq \sum_{a \in A} t_a(x_a, z_a) v_a \quad \forall v \quad \text{feasible}$$

Variational inequality problem for traffic equilibrium

$$N x^k = d^k \quad \forall k \in \Omega$$

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$$N x^k = d^k \quad \forall k \in \Omega$$

$$x, z \in R^+$$

Variational inequality problem for traffic equilibrium

Solution: MIP approximate reformulation

Robust Network Design Problem with Traffic Equilibrium

We assume that **demand** belongs to given uncertainty set

$$\min \gamma$$

$$\text{s.a} \sum_{a \in A} g_a(z_a) \leq B$$

$$\forall\, d_k \in \mathcal{U}_{d_k} \text{exists } x : \begin{cases} \sum_{a \in A} t_a(x_a, z_a) x_a \leq \gamma \\ \text{s.a} \quad \sum_{a \in A} t_a(x_a, z_a) x_a \leq \sum_{a \in A} t_a(x_a, z_a) v_a \quad \forall v \quad \text{feasible} \\ N x^k = d^k \quad \forall k \in \Omega \end{cases}$$

$$x, z \in R^+$$

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