Well-posedness of deterministic bilevel games through a general stochastic approach

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Ill-posedness

We all have noted that

$$(P) = \begin{cases} \min\limits_{x} & \theta(x, y) \\ \text{s.t.} & x \in X, \\ & y \in S(x). \end{cases}$$

is ill-posed.

The reason: If $S : X \rightrightarrows Y$ is not single-valued, there is an ambiguity on how to “choose” the follower’s reaction $y \in S(x)$.

Two extreme approaches

$$\underbrace{\begin{cases} \min\limits_{x} \color{red}{\min\limits_{y}} & \theta(x,y) \\ \text{s.t.} & x \in X, \\ & y \in S(x). \end{cases}}_{\text{Optimistic}} \quad \xleftrightarrow{\text{spectrum}} \quad \underbrace{\begin{cases} \min\limits_{x} \color{red}{\max\limits_{y}} & \theta(x,y) \\ \text{s.t.} & x \in X, \\ & y \in S(x). \end{cases}}_{\text{Pessimistic}}$$

- For the optimistic approach, the leader **believes** that the follower will help him (naive).
- For the pessimistic approach, the leader **believes** that the follower is against him (risk-averse).

The leader believes...

We define a **belief** of the leader as a mapping $\beta : X \to \mathscr{P}(Y)$ such that, for each leader's decision $x \in X$:

1. β assigns **a probability distribution** $\beta(x) = \beta_x$ over Y.
2. β_x concentrates on $S(x)$, that is, $\beta_x(S(x)) = 1$.

An example just in the middle: The indifferent approach.

$\forall x \in X$, β_x is the uniform distribution over $S(x)$.

Stochastic approach

If the leader has a belief β of how the follower will react, then he must solve

$$P(\beta) = \begin{cases} \min\limits_{x} & \mathbb{E}_{\beta_x}(\theta(x,\cdot)) := \int\limits_{Y} \theta(x,y) d\beta_x(y) \\ \text{s.t.} & x \in X. \end{cases}$$

Please, check our submitted article for existence of solutions (even for multiple leaders):

D. Salas and A. Svensson. "Well-posedness of deterministic bilevel games through a general stochastic approach". In: *arXiv:2010.05368* (2020). (Submitted)

There is so much more to explore here!