A Fast Non-Regularized Numerical Algorithm for Solving Bilevel Denoising Problems

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Denoising Models

Denoising Models

Let $f \in \mathbb{R}^n$ be a noisy image, the scale dependant ROF ([Rudin Osher and Fatemi - 92]) denoising model in finite dimension is defined as

$$\min_{u \in \mathbb{R}^n} \mathcal{J}(u) = \phi(u, f) + \sum_{j=1}^{n} \alpha_j \|(\mathbb{K}u)_j\|_2.$$

Where $\|\mathbb{K} \cdot \|_{2,1}$ is the *Isotropic Total Variation Operator*

$$\|\mathbb{K}u\|_{2,1} = \sum_{j=1}^{n} \|(\mathbb{K}u)_j\|_2 = \sum_{j=1}^{n} \sqrt{(\mathbb{K}_x u)_j^2 + (\mathbb{K}_y u)_j^2}. \tag{1}$$

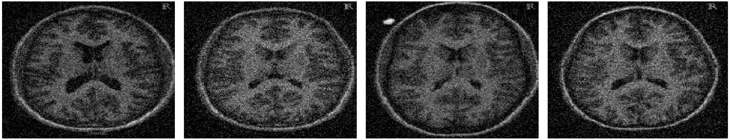
How to pick the best regularization parameter?

Clean $\alpha = 0.05$ $\alpha = 0.13$

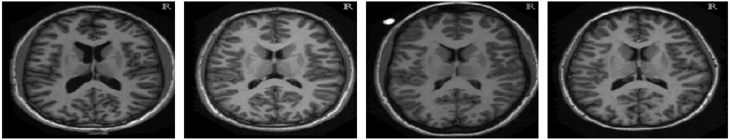
Learning from training sets

Image denoising training sets such as

Low resolution MRI scan

f_k  $\cdots$

High resolution MRI scan

u_k  $\cdots$

Simulated data from OASIS online database. Arridge, Kaipio, Kolehmainen, Schweiger, Somersalo, Tarvainen, Vauhkonen '06; Benning, Gladden, Holland, CBS,Valkonen '14

Bilevel Image Reconstruction Model

Assumptions

Training set of pairs $(f_k, u_k),\ k = 1, \ldots, N$ with

- f_k imperfect data
- $\bar{u}_k$ represent the ground truth

Vectorial Denoising Model

Let $(f_k, \bar{u}_k)_{i=1,\ldots,N}$ a training set, the optimal parameter $\alpha \in \mathbb{R}^n$ is the solution of the following bilevel problem

$$\min_{\alpha \in \mathbb{R}^n} J(u) := \sum_{k=1}^{N} \|\bar{u}_k - u_k\|_2^2 \tag{2a}$$

$$s.t.\ u_k = \arg\min_{u \in \mathbb{R}^n} \phi(u, f_k) + \sum_{j=1}^{n} \alpha_j \|(\mathbb{K}u)_j\|_2, \tag{2b}$$

Lower Level Problem

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Variational Inequality

A necessary and sufficient condition for problem the lower level problem is given by the following variational inequality of the second kind:

$$\langle \phi'(u), v - u \rangle + \sum_{j=1}^{n} \alpha_j \|(\mathbb{K}v)_j\|_2 - \sum_{j=1}^{n} \alpha_j \|(\mathbb{K}u)_j\|_2 \geq 0, \; \forall v \in \mathbb{R}^n.$$

Generalized equation

A necessary and sufficient condition for problem the lower level problem is given by the following generalized equation:

$$0 \in \phi'(u) + G(\alpha, u),$$

where

$$G(\alpha, u) := \{\mathbb{K}^\top q : \begin{cases} q_j = \alpha_j \frac{(\mathbb{K}u)_j}{\|(\mathbb{K}u)_j\|} \text{ if } (\mathbb{K}u)_j \neq 0, \\ \|q_j\| \leq \alpha_j \text{ else} \end{cases} \}.$$

Challenges

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1. Bouligand differentiability of the solution operator $S : \mathbb{R}^n_+ \mapsto \mathbb{R}^n$.
2. Classical constraint qualification conditions are violated.
3. Characterize first order optimality conditions.
4. Derive a non-smooth trust-region algorithm to solve the non-regularized problem.

Numerical Results

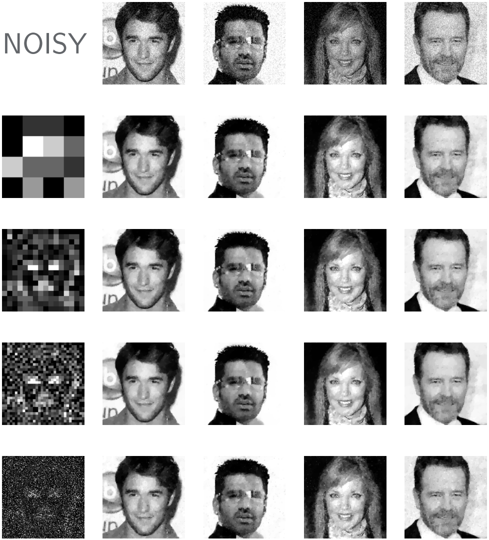

Figure: Patch-Dependent and Scale-Dependent Regularization Parameter

THANKS!

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