The bilevel continuous knapsack problem with uncertain follower's objective

Christoph Buchheim and *Dorothee Henke*

TU Dortmund University

Bilevel optimization under uncertainty

Robust Bilevel Optimization

$$
\begin{aligned}
\max_{y} \quad & \min_{\xi \in U} \quad f(y,z) \\
\text{s.t.} \quad & (y,z) \in Y \\
& z \in \arg\max_{z} \quad g_{\xi}(y,z) \\
& \qquad \text{s.t.} \qquad z \in Z(y)
\end{aligned}
$$

- leader doesn't know follower's problem exactly
- **given:** uncertainty set U of possible scenarios
- **goal:** optimize worst case

- three decision makers:
  1. leader
  2. adversary
  3. follower

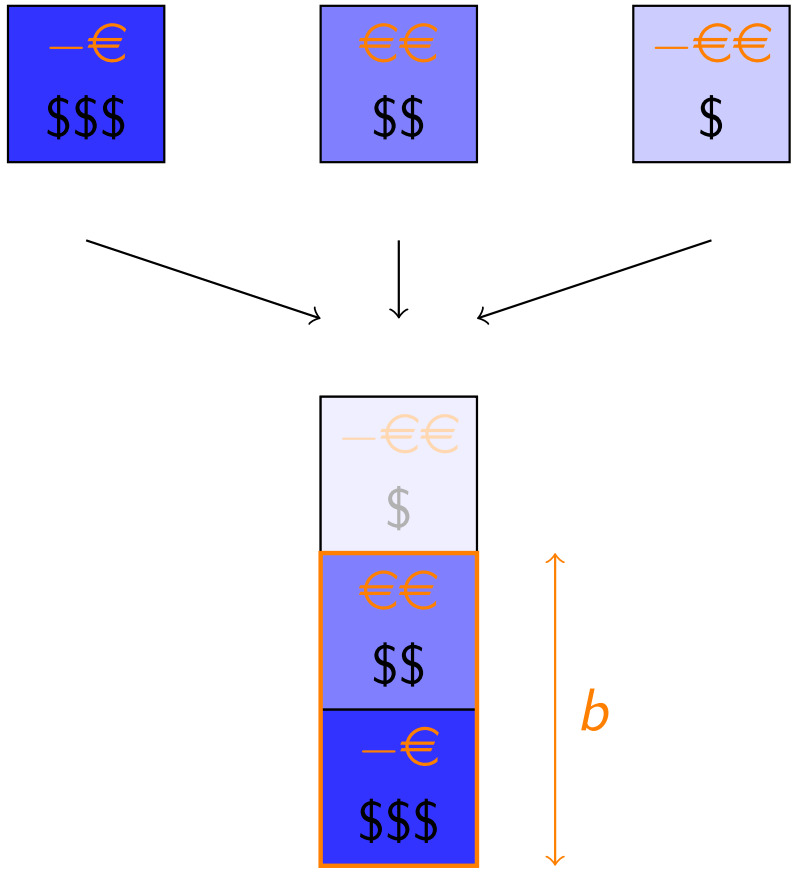
Complexity Question

How much harder can bilevel problems become with robustness?

An easy bilevel optimization problem

Bilevel Continuous Knapsack

$$\begin{aligned} \max_{b \in [b^-, b^+]} \quad & d^\top x \\ \text{s. t.} \quad & x \in \arg\max_x \quad c^\top x \\ & \qquad \text{s. t.} \quad a^\top x \leq b \\ & \qquad\qquad 0 \leq x \leq 1 \end{aligned}$$

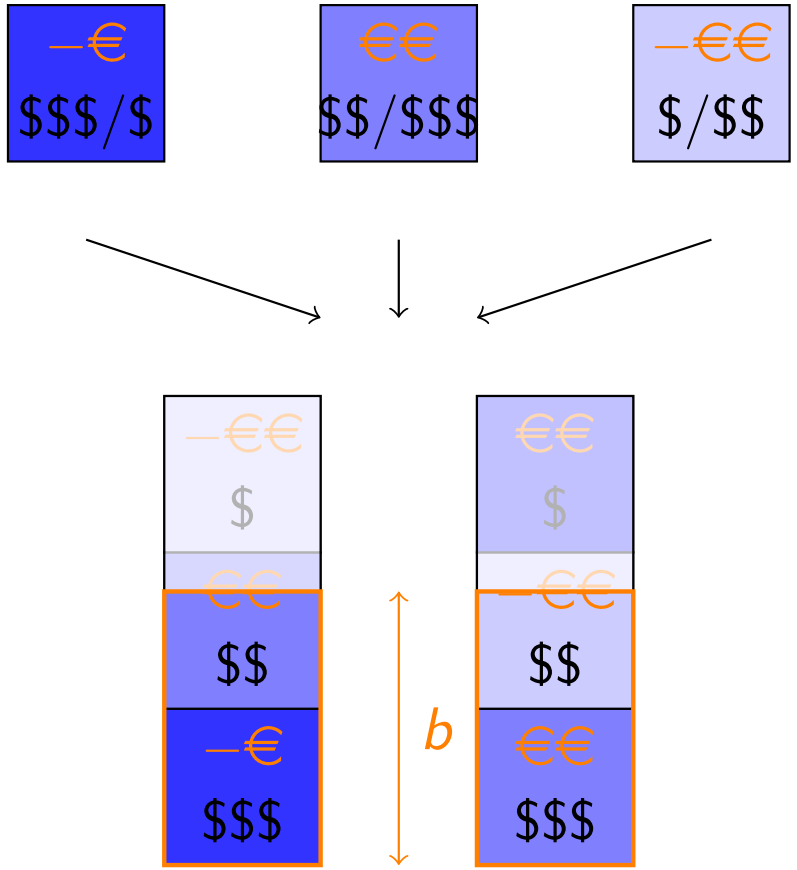


This problem can be solved in polynomial time.

Adding robustness

Robust Bilevel Continuous Knapsack

$$\max_{b\in[b^-,b^+]} \quad \min_{c\in U} \quad d^\top x$$
$$\text{s. t.} \quad x \in \arg\max_{x} \quad c^\top x$$
$$\text{s. t.} \quad a^\top x \leq b$$
$$0 \leq x \leq 1$$



Some results

- For a finite set U, this problem can be solved in polynomial time.
- For a polytope U, this problem is NP-hard.

More details

https://arxiv.org/abs/1903.02810

Thank you!