Aircraft deconfliction via Bilevel Programming

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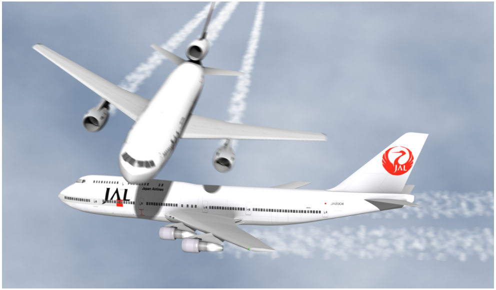
Aircraft deconfliction



- aircraft speed changes (SRADP)
- heading angle changes (HACADP)[1]

[1] M. Cerulli, C. D'Ambrosio, L. Liberti, M. Pelegrín: Detecting and solving aircraft conflicts using bilevel programming, *submitted paper*

Sets, parameters, variables

1. **Sets:**
   - $A = \{1, \ldots, n\}$ is the set of aircraft
   - $K = \{1, \ldots, k_{max}\}$ is the set of dimension indices

2. **Parameters:**
   - T is the length of the time horizon taken into account [hours]
   - **d** is the minimum required safety distance between a pair of aircraft [Nautical Miles NM]
   - x_{ik}^0 is the k-th component of the initial position of aircraft i
   - v_i is the initial speed of aircraft i [NM/h]
   - u_{ik} is the k-th component of the direction of aircraft i
   - q_i^{min} and q_i^{max} are the bounds on the speed modification ratios for each aircraft i

3. **Variables:**
   - q_i is the ratio of the implemented speed w.r.t. the initally planned speed of i: $q_i = 1$ if the speed remains constant, $q_i > 1$ if it is increased, $q_i < 1$ if it is decreased. (q_i is constant in the time horizon considered)

Bilevel formulation

$$\min_{q,t} \quad \sum_{i \in A} (q_i - 1)^2$$

$$\forall i \in A \quad q_i^{\min} \leq q_i \leq q_i^{\max}$$

$$\forall i < j \in A \quad \min_{t_{ij} \in [0,T]} \sum_{k \in K} \left((x_{ik}^0 - x_{jk}^0) + t_{ij} (q_i v_i u_{ik} - q_j v_j u_{jk}) \right)^2 \geq d^2$$

- Single level reformulation (e.g. using KKT or Dual of the lower level)
- Cut generation algorithm

Cut generation algorithm for SRADP

1. Let $h = 1$. Initialize the relaxation R_h of the bilevel program, obtained by considering **the upper-level problem only**.
2. Solve R_h to obtain the optimal solution q^*.
3. For each aircraft pair (i, j), compute the instant $\tau_{ij}^h \in [0, T]$ as

$$\tau_{ij}^h = \min\left\{\max\left\{0, -\frac{\sum_{k\in K}(x_{ik}^0 - x_{jk}^0)(q_i^* v_i u_{ik} - q_j^* v_j u_{jk})}{\sum_{k\in K}(q_i^* v_i u_{ik} - q_j^* v_j u_{jk})^2}\right\}, T\right\}.$$

4. If $\sum_{k\in K}((x_{ik}^0 - x_{jk}^0) + \tau_{ij}^h(q_i^* v_i u_{ik} - q_j^* v_j u_{jk}))^2 \geq d^2 \quad \forall i < j \in A$ the algorithm terminates and q^* is the optimal bilevel solution. Else, for each pair (i, j) violating the inequality, define R_{h+1} as R_h with the adjoined inequality:

$$\sum_{k\in K}((x_{ik}^0 - x_{jk}^0) + \tau_{ij}^h(q_i v_i u_{ik} - q_j v_j u_{jk}))^2 \geq d^2.$$

5. Put $h := h + 1$ and go back to 2.

Thanks for your attention!