Bi-level Programming and Solution Techniques for Allocation of Healthcare Funds

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Introduction to the Problem

- Remarkable progress towards Prevention and Treatment of Diseases world-wide in last two decades
- Not only due to technological development in medical sciences but also due to the the aid money for getting the resources
- Aid money: rich countries and philanthropic organizations - central donor agencies
- These Funds are limited (also, issues like rising population and number of new diseases emerging)
- Problem: Allocation of these Healthcare Funds available with central agency (DONOR)

Allocation of Healthcare Funds

- Conventional Approach: Cost-effective analysis
  Donor prioritizes projects with higher Profit/Cost ratio

- Disadvantages:
  - Under-allocation of resources
  - Crowding out of indigenous financing of projects
  - No consideration if the donor differs project valuations from the country

- Proposed Approach: Instead of funding an entire project, DONOR gives subsidy to any project in the pipe-line of recipient Country (COUNTRY)[1]

[1] Alec Morton, Ashwin Arulselvan, and Ranjeeta Thomas. "Allocation rules for global donors". In: *Journal of Health Economics* 58 (2018), pp. 67 –75. ISSN: 0167-6296.

Background to Proposed Approach

- COUNTRY has a budget for all its projects of national importance
- Hence, healthcare projects in a Country not only compete for funds within but also with other projects like education and infrastructure
- COUNTRY has a margin for selection of healthcare projects
- Projects with higher Profit/Cost ratio than the margin are selected for funding from the national budget.

Proposed Approach

DONOR subsidizes the projects that are just cost-ineffective to make them eligible to be funded by the COUNTRY[2]

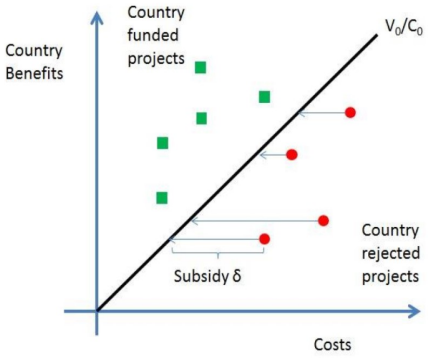

Figure: Healthcare Projects and Funding based on the line of Marginal Cost-effectiveness

[2] Alec Morton, Ashwin Arulselvan, and Ranjeeta Thomas. "Allocation rules for global donors". In: *Journal of Health Economics* 58 (2018), pp. 67 –75. ISSN: 0167-6296.

Potential Advantages: Efficient allocation of the available funds and Sustainable aid

Modelling the Problem:

- Analogy of Donor-Country captured by the leader-follower strategic games
- These games are modelled as Bi-level programs
- Upper level is DONOR and lower level is donation-receiving COUNTRY

BKP for Allocation of Healthcare Funds

DONOR:

$$\text{maximize} \quad \sum_{i \in I} w_i x_i \tag{1a}$$

$$\text{subject to} \quad \sum_{i \in I} c_i \delta_i' \leq B_{donor} \tag{1b}$$

$$\delta_i' \leq x_i \qquad \forall i \in I \tag{1c}$$

$$\delta_i' \in [0,1] \qquad \forall i \in I \tag{1d}$$

$$x \in \ argmax(COUNTRY) \tag{1e}$$

COUNTRY:

$$\text{maximize} \quad \sum_{i \in I} v_i x_i + v_0 x_0 \tag{2a}$$

$$\text{subject to} \quad \sum_{i \in I} c_i x_i + c_0 x_0 \leq B_{country} + \sum_{i \in I} c_i \delta_i' \tag{2b}$$

$$x_i \in \{0,1\} \qquad \forall i \in I \tag{2c}$$

$$x_0 \in [0,1] \tag{2d}$$

n: no. HC projects in set I
w_i: donor valuation for HC project i
v_i: country valuation for HC project i
c_i: cost of HC project i
v_0: valuation of external project
c_0: cost of external project i
B_{donor}: donor budget
$B_{country}$: country budget

Decision variables:
δ_i': proportion of cost c_i for HC project i that gets subsidised from the donor budget
x_i: indicates if HC project i is funded
x_0: proportion of funding for external project by the country

Bi-level Knapsack Problems

- Special class of MIBLPs where the upper level problem decides budget of the knapsack and lower level problem has binary variables to select items in the knapsack
- Depending on the structure of the problem, upper and lower variables may or may not be there in the other level where they belong to
- Complexity of BKPs[3]
- Enumeration algorithm[4]
- Solution techniques for other MIBLPs: Reformulation, DP and Branch-And-Cut Algorithms

[3] Alberto Caprara et al. "A Complexity and Approximability Study of the Bilevel Knapsack Problem". In: *Proceedings of the 16th International Conference on Integer Programming and Combinatorial Optimization*. IPCO'13. Berlin, Heidelberg: Springer-Verlag, 2013, pp. 98–109.

[4] S. Dempe and K. Richter. "Bilevel Programming With Knapsack Constraints". In: *Central European Journal of Operations Research* 8.2 (2000), pp. 93–107.

Algorithm to solve proposed BKP

- The upper level variables of subsidy from DONOR appear in the lower level
- The lower level has both continuous and discrete variables. The discrete lower level variable appears in the upper level objective function
- Complex structure to solve with available solution methods
- Proposed Reformulation Algorithm: The dual of lower level problem is added as a set of constraints in the upper problem
- The set of feasible points to relaxed upper problem but infeasible to lower level problem can be removed using these constraints

Computational Experiments I

- Language: Python
- Solver: Gurobi
- Data:
  - Classes of Healthcare Projects: 3
    - High Profit/Cost
    - Medium Profit/Cost
    - Low Profit/Cost
  - Input Parameters for HC projects:
    - Number of projects in each class
    - Lower and Upper limit for Country and Donor Valuation, Project costs in each class
    - Random Number generated between Upper and Lower limits for above
  - Input Parameters for External projects:
    - Lower and Upper limit for Project cost
    - Random Number generated between Upper and Lower limits for above
    - Country Val of Ext Project = Project Cost * Gamma * Avg Profit/Cost of either of the 3 HC projects classes

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Thank You

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