Transmission Planning in an Imperfectly Competitive Power Sector with Environmental Externalities

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14 May 2024

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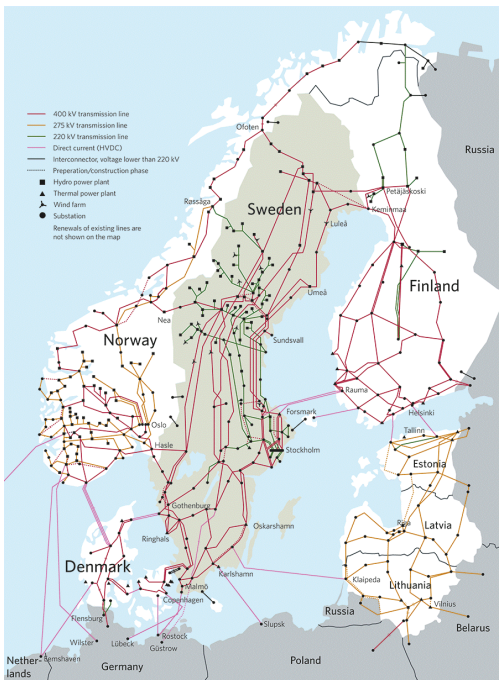
Introduction

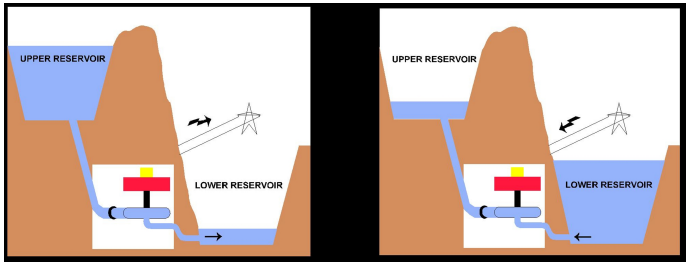
The Nordic Grid: Facilitating the Green Deal

- **Electricity market is efficient (Amundsen and Bergman, 2006)**
  - **Hydro reservoirs and regional integration dilute market power**
  - **Market design: forward contracts and transparent information**
- **Transmission upgrades required to integrate variable renewable energy (VRE) as part of ambitious climate targets**



Svenska kraftnät (2023)

Anticipate Market Imperfections and Environmental Externalities in Transmission Planning



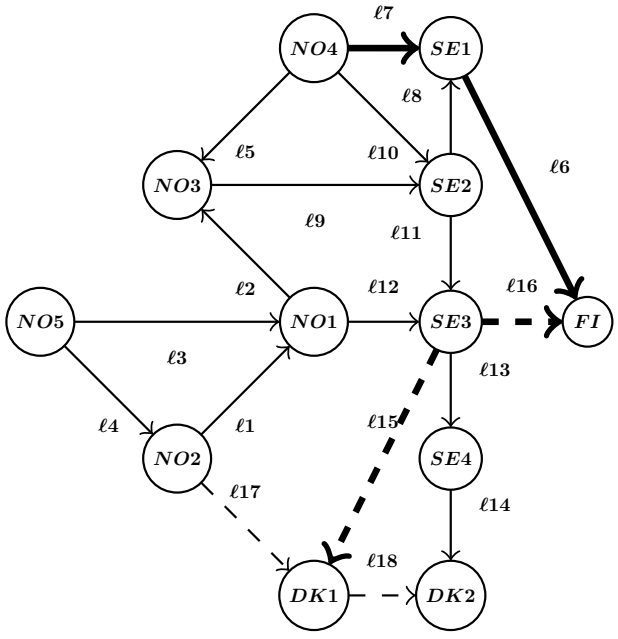
https://www.ctc-n.org/collection/climatetechwiki

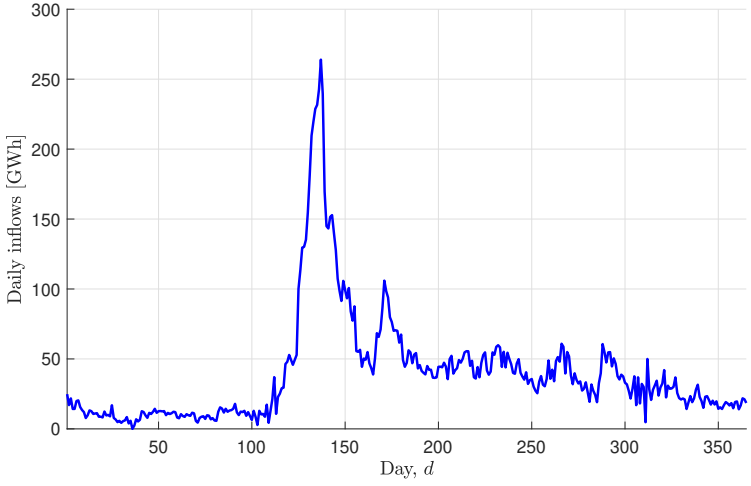
Ewald et al. (2022)

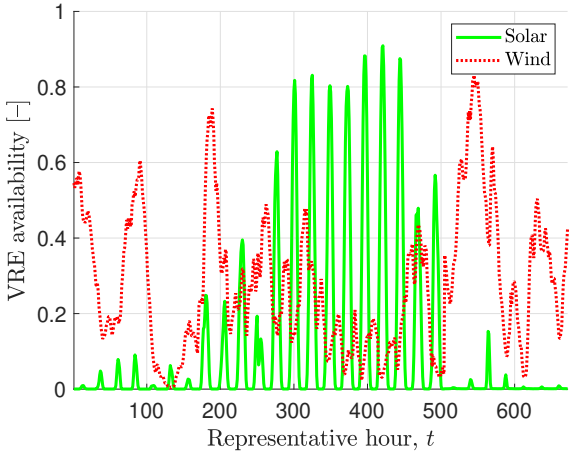
Generation-and-Transmission-Expansion Planning (GTEP)

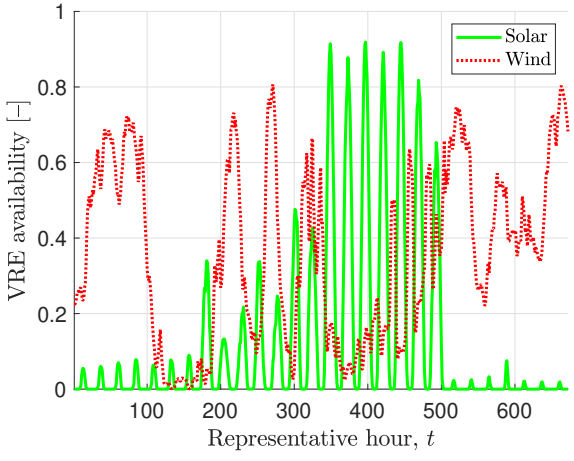
- Socially optimal transmission plan equates marginal benefits and marginal costs of transmission expansion (Garver, 1970)
- Engineering/OR literature focuses on spatio-temporal and technical attributes of the power system
  - Optimal regional GTEP (Rodríguez-Sarasty et al., 2021)
  - Uncertainty incorporated via robust optimisation (García-Cerezo et al., 2022; Moreira et al., 2021; Rintamäki et al., 2024)
  - Departures from central planning to address VRE adoption (Baringo and Conejo, 2012), RPS targets (Maurovich-Horvat et al., 2015), and energy storage (González-Romero et al., 2021)
- Environmental economics emphasises tradeoffs explicitly (Hobbs, 2012)
  - Stylised framework with closed-form solutions (Barnett, 1980)
  - Carbon taxation actually increases emissions in the presence of market power and congested transmission line (Downward, 2010)
  - Transmission expansion requires only partial internalisation of damage cost under Cournot oligopoly (Siddiqui et al., 2019)

Stackelberg Model of Nord Pool with Spatio-Temporal Details





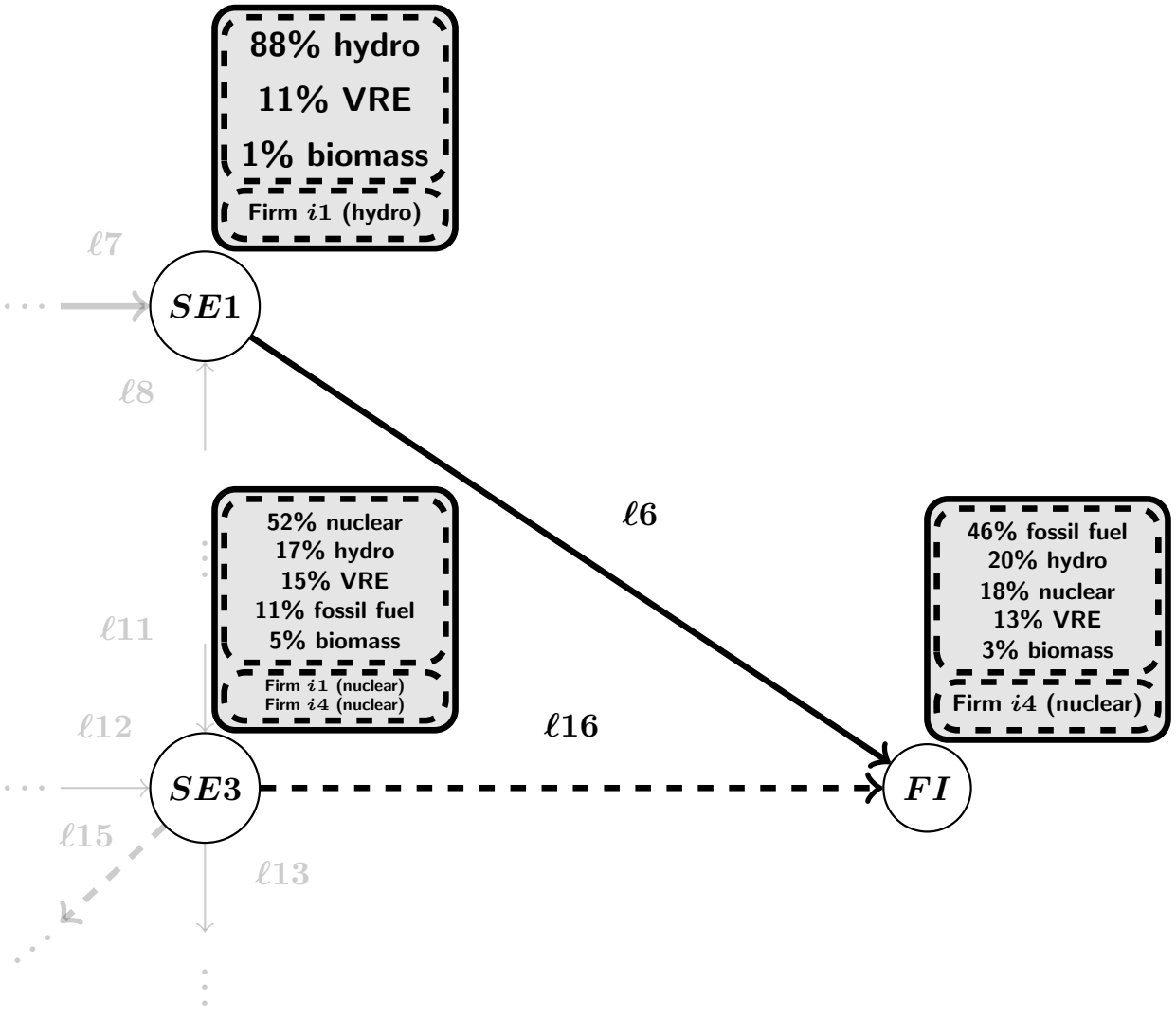




Research Objective and Findings

- Bi-level GTEP (Gabriel et al., 2013) for the Nordic region
  1. What is the first-best transmission plan?
  2. How does imperfect competition affect transmission planning under full internalisation of the damage cost of CO_2 emissions?
  3. How should transmission planning be adapted under perfect/imperfect competition and partial internalisation of the damage cost of CO_2 emissions?
- RQ 1: expand $\ell 16$ to enable better sharing of nuclear resources between zones and limit some VRE adoption at FI
- RQ 2: nuclear withholding bolsters transmission expansion (both $\ell 6$ and $\ell 16$), while hydro arbitrage reduces some flows as high prices and wind availability boost VRE adoption
- RQ 3: under perfect competition, modify first-best transmission plan to increase VRE investment
  - Less nuclear withholding diminishes transmission expansion
  - Less hydro arbitrage weakens incentives for VRE adoption, thereby prompting more transmission expansion

Network Map of Zones $SE1$, $SE3$, and FI with 2018 Generation-Capacity Mixes and Strategic Firms

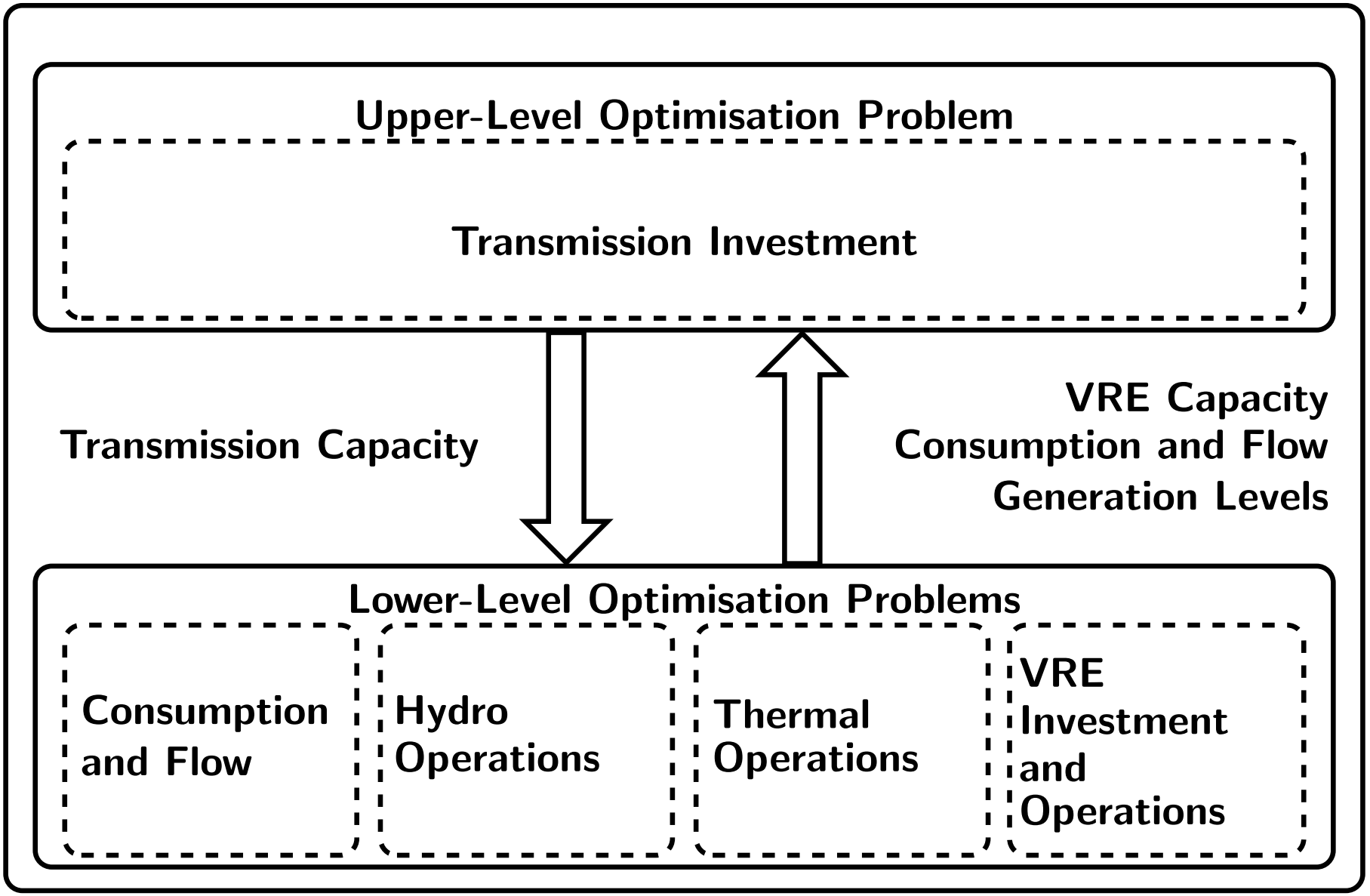


Problem Formulation

Assumptions

- Linear inverse demand, $D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} q_{n,t}, \forall n \in \mathcal{N}, t \in \mathcal{T}$
- Firm $i \in \mathcal{I}$ may own thermal, VRE, and hydro capacity
  - Thermal plant $u \in \mathcal{U}_{i,n}$ has capacity $\overline{G}_{i,n,u}$, operating cost $C_{i,n,t,u}$, CO_2 emission rate $P_{i,n,u}$, and ramp rates R^{up}_u, R^{down}_u
  - VRE $e \in \mathcal{E}_{i,n}$'s output $g^e_{i,n,t}$ depends on availability factor $A^e_{n,t}$
  - Hydro unit $w \in \mathcal{W}_{i,n}$ has reservoir limits $\overline{R}_{i,n,w}, \underline{R}_{i,n,w}$, natural inflows $I_{i,n,t,w}$, and generation capacity $\overline{Y}_{i,n,w}$
  - Capacities, $a_{i,n,u}, a^e_{i,n}, a_{i,n,w}$, given fixed O&M costs, while VRE capacity expansion $b^e_{i,n}$ at amortised investment cost
- Surplus-maximising ISO manages transmission flows, $\hat{f}_{\ell,t}$, by controlling voltage angles, $v_{n,t}$, subject to thermal limits (Hobbs, 2001; Sauma and Oren, 2007; Tanaka, 2009)
- Welfare-maximising TSO selects discrete capacity sizes $j \in \mathcal{J}_\ell$ for line $\ell \in \mathcal{L}$ via binary variables, $x_{j,\ell}$, while taking social-cost rate of CO_2 emissions, S, and internalisation rate, H, as given

Bi-Level Model



Problem Formulation

Maximise Gross Consumer Surplus
- Cost of Thermal Generation
- Amortised Cost of Generation Investment
- Amortised Fixed O&M Cost
- Amortised Cost of Transmission Investment
- Social Cost of CO_2 Emissions (1)

s.t.

Discrete Transmission Capacity (2)–(3)

Maximise Gross Consumer Surplus
- Cost of Thermal Generation
- Amortised Cost of Generation Investment
- Amortised Fixed O&M Cost
- Extended Cost
- Private Cost of CO_2 Emissions (56)

s.t.

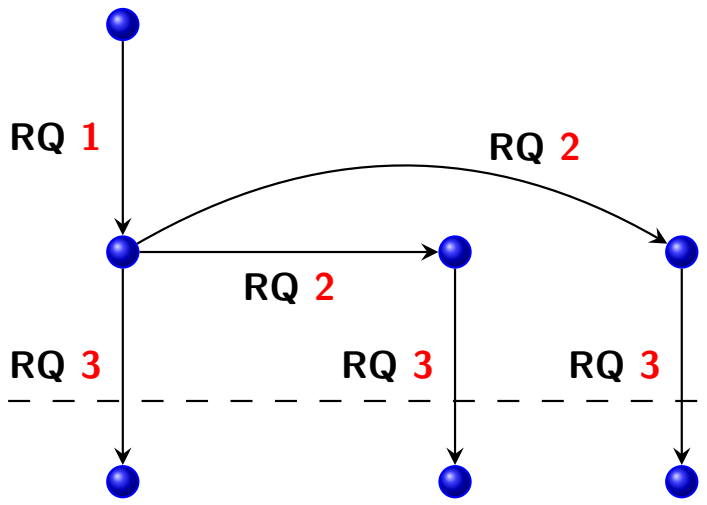
Energy-Balance Constraint (5)
Capacity Limit on Transmission (6)–(7)
DC Load-Flow Constraints (8)–(9)
Constraints on Thermal Generation (11)–(13)
Constraints on VRE Generation (14)–(15)
Constraints on Hydro Generation (16)–(20)
Annual Hydro Regulation (21)

Numerical Examples

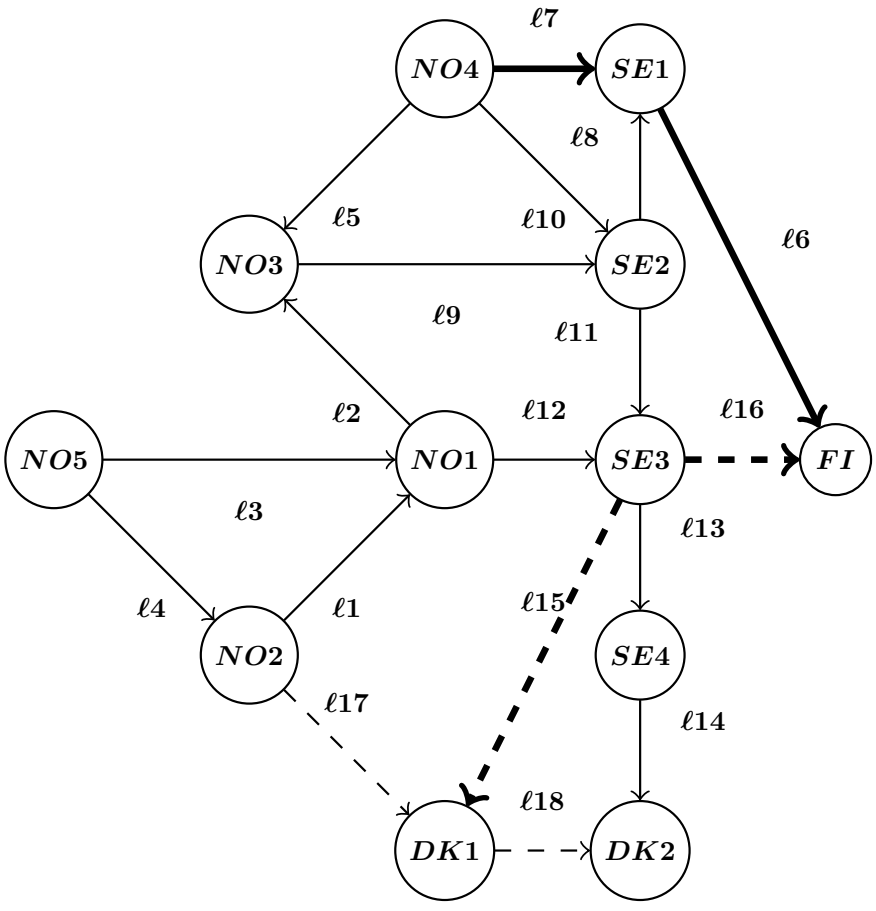
Cases, Scenarios, and Regimes

- Cases reflect degree of competition
  - Perfect competition (PC): all firms are price takers
  - Cournot oligopoly in thermal generation (COG): firms with large capacities withhold generation from nuclear plants
  - Cournot oligopoly in reservoirs (COR): firms with strategic reservoirs exercise market power in hydro-reservoir generation
- Scenarios capture plausible futures
  - Base2018: 2018 generation and transmission capacities with $S = 15$
  - FutureC: same as Base2018 except that $S = 100$
  - FutureCV: same as FutureC except that VRE expansion is allowed by firms at nodes where they own VRE capacity
  - FutureCVT: same as FutureCV except that transmission expansion is allowed on selected lines
- Regimes indicate political sentiment
  - Complete carbon pricing: $H = 1$
  - Incomplete carbon pricing: $H = 0.15$

Design of Experiment

Regime	Scenario \ Case	PC COG COR
$H = 1$	Base2018	
	FutureCVT	
$H = 0.15$	FutureCVT	

Transmission Network, Line Capacities (MW), and Line Susceptances (S)



Line	$\overline{K}_{j0,\ell}$	$B_{j0,\ell}$
$\ell 1$	3500	1628
$\ell 2$	500	898
$\ell 3$	3900	1275
$\ell 4$	600	1346
$\ell 5$	1200	317
$\ell 6$	1500	460
$\ell 7$	700	688
$\ell 8$	3300	798
$\ell 9$	600	981
$\ell 10$	250	302
$\ell 11$	7300	1081
$\ell 12$	2145	822
$\ell 13$	5400	1226
$\ell 14$	1300	1578
$\ell 15$	680	–
$\ell 16$	1200	–
$\ell 17$	1632	–
$\ell 18$	590	–

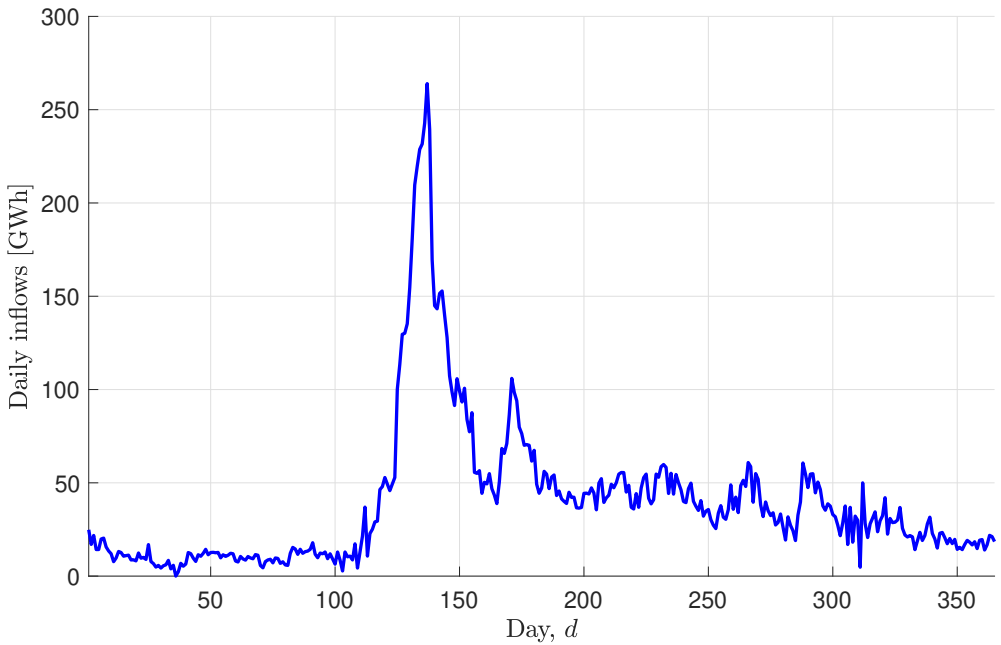
Congestion Analysis and Candidate Lines

Line	$\ell 4$	$\ell 16$	$\ell 2$	$\ell 7$	$\ell 6$	$\ell 15$	$\ell 17$	$\ell 18$
Proportion of Hours Congested	0.923	0.732	0.686	0.646	0.570	0.525	0.488	0.441

Line	$\ell 6$	$\ell 7$	$\ell 15$	$\ell 16$	$\ell 18$
$B_{j,\ell^{\mathrm{AC}}}$	141	213	–	–	–
$C^{\mathrm{trn}}_{j,\ell}$	49.44	37.76	45.60	31.92	12.48

Hydro Production

- Hydro units are categorised as follows:
  - Run-of-river (ROR)
  - Non-strategic reservoirs (NRS)
  - Non-strategic pumped hydro (NPH)
  - Strategic reservoirs (SRS)
  - Strategic pumped hydro (SPH)
- Inflows estimated based on the annual production of the units



Thermal Generation Costs, Emission Rates, and Ramp Rates

Unit	$C_{i,n,t,u}$	$P_{i,n,u}$	Ramp Rate (–)
Coal $u1$	32	0.83	0.2
Gas $u2$	65	0.50	0.5
CCGT $u3$	48	0.37	0.5
Oil $u4$	67	0.72	0.7
Biomass $u5$	59	0.00	0.2
Nuclear $u6$	21	0.00	0.1
Peat $u7$	22	1.09	0.1
Waste $u8$	22	0.94	0.1
CHP Coal $u9$	37	0.83	0.1
CHP Waste $u10$	22	0.94	0.1
CHP Gas $u11$	57	0.50	0.1
CHP Oil $u12$	33	0.72	0.1
CHP Peat $u13$	22	1.09	0.1
CHP Biomass $u14$	27	0.00	0.1

Firms' Installed Capacities by Node and Unit (GW)

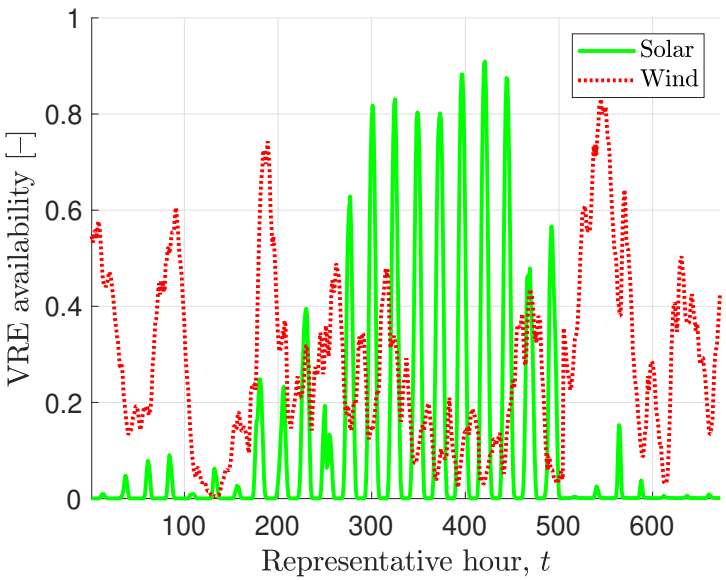
Node	Firm	$u1$	$u2$	$u3$	$u4$	$u5$	$u6$	$u7$	$u8$	$u9$	$u10$	$u11$	$u12$	$u13$	$u14$	Wind	Solar	Hydro
$SE1-SE4$	$i1$	–	–	–	–	–	4.9	–	–	–	0.1	–	–	–	0.1	0.3	–	7.5
	$i2$	–	–	–	–	–	0.7	–	–	–	–	–	–	–	0.1	0.2	–	–
	$i3$	–	–	–	–	–	0.8	–	–	–	–	–	–	–	–	–	–	–
	$i4$	–	–	–	–	–	1.4	–	–	–	0.1	–	–	0.2	0.1	0.1	–	3.5
	$i10$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.1	–	1.1
	$i16$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2.2
	$i17$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.5
	$i18$	–	0.4	–	1.8	–	–	–	–	–	–	–	–	–	0.1	5.6	0.2	1.6
FI	$i4$	0.3	–	–	–	–	1.5	–	–	0.1	–	0.3	–	–	0.1	–	–	1.5
	$i6$	0.3	–	–	–	–	1.0	–	–	–	–	–	–	–	0.4	–	–	0.4
	$i7$	–	–	–	0.1	–	–	–	–	0.2	–	0.7	–	0.2	–	–	–	–
	$i8$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.4
	$i19$	–	–	–	1.2	–	0.3	–	–	0.7	0.2	0.9	0.1	2.1	–	1.9	0.2	0.7
$DK1-DK2$	$i1$	0.4	–	–	–	–	–	–	–	–	–	–	–	–	–	0.8	–	–
	$i9$	1.4	0.7	1.2	–	0.1	–	–	–	–	–	0.3	–	–	1.6	0.4	–	–
	$i20$	0.4	–	0.3	–	–	–	–	–	–	–	–	–	–	–	4.4	0.9	–
$NO1-NO5$	$i2$	–	0.2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	$i4$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.1	–	–
	$i10$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.2	–	9.5
	$i11$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2.3
	$i12$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	3.9
	$i13$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2.0
	$i14$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1.8
	$i15$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	4.4
	$i21$	–	0.8	0.1	–	–	–	–	0.1	–	–	–	–	–	–	1.7	0.1	12.4

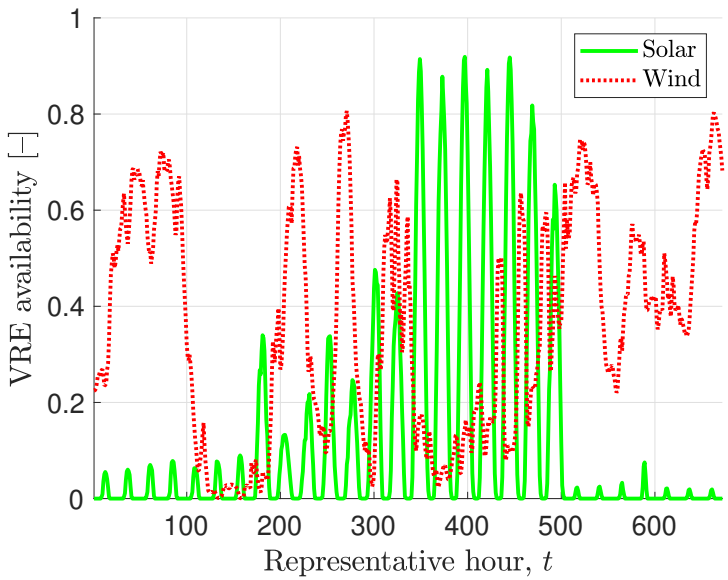
Firms' Hydro Reservoirs by Node and Type (GWh)

Node	Firm	SRS	NRS	NPH	SPH
$SE1 - SE4$	$i1$	12210	4668		
	$i4$		5952		
	$i10$		2533		
	$i16$		4105		
	$i17$		1626		
	$i18$		2457		
FI	$i5$		1268		
	$i7$		4262		
$NO1 - NO5$	$i10$	17707	15508	2823	
	$i11$	99	5406		
	$i12$		4328	681	
	$i13$	276	4506	95	
	$i14$	2016	1331		130
	$i15$	4646	4746		421
	$i21$		26234	701	

VRE and Demand

- **Most existing VRE capacity in Denmark**
- **Demand: construct nodal inverse-demand functions via estimates of own-price elasticities (Debia et al., 2021)**
- **Time resolution: representative weeks for each of the four seasons**





Calibration

- Total generation: 398 TWh (compared to 398 TWh in 2018)
- Total net-hydro generation: 212 TWh (compared to 213 TWh in 2018)
- Average electricity prices: €41.55/MWh under PC, €75.10/MWh under COG, and €44.25/MWh under COR (compared to €42.04/MWh in the representative weeks)
- Total CO_2 emissions: 28.28 Mt (compared to 35.1 Mt for both power and heat generation in 2017)
- Seasonal reservoir levels under PC and COR typically follow the observed patterns for 2018 (Hassanzadeh Moghimi et al., 2023)

Summary Results in the Base2018 Scenario (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.942	137.474	138.843
Consumer Surplus	128.144	115.257	126.898
Producer Surplus	10.379	21.241	11.377
Merchandising Surplus	0.420	0.976	0.568
Government Revenue	0.424	0.857	0.427
CO_2 Damage Cost	0.424	0.857	0.427
CO_2 Emissions (Mt)	28.275	57.121	28.470
Firm $i1$'s Surplus	1.467	2.517	1.637
Average Price (€/MWh)	41.548	75.101	44.248

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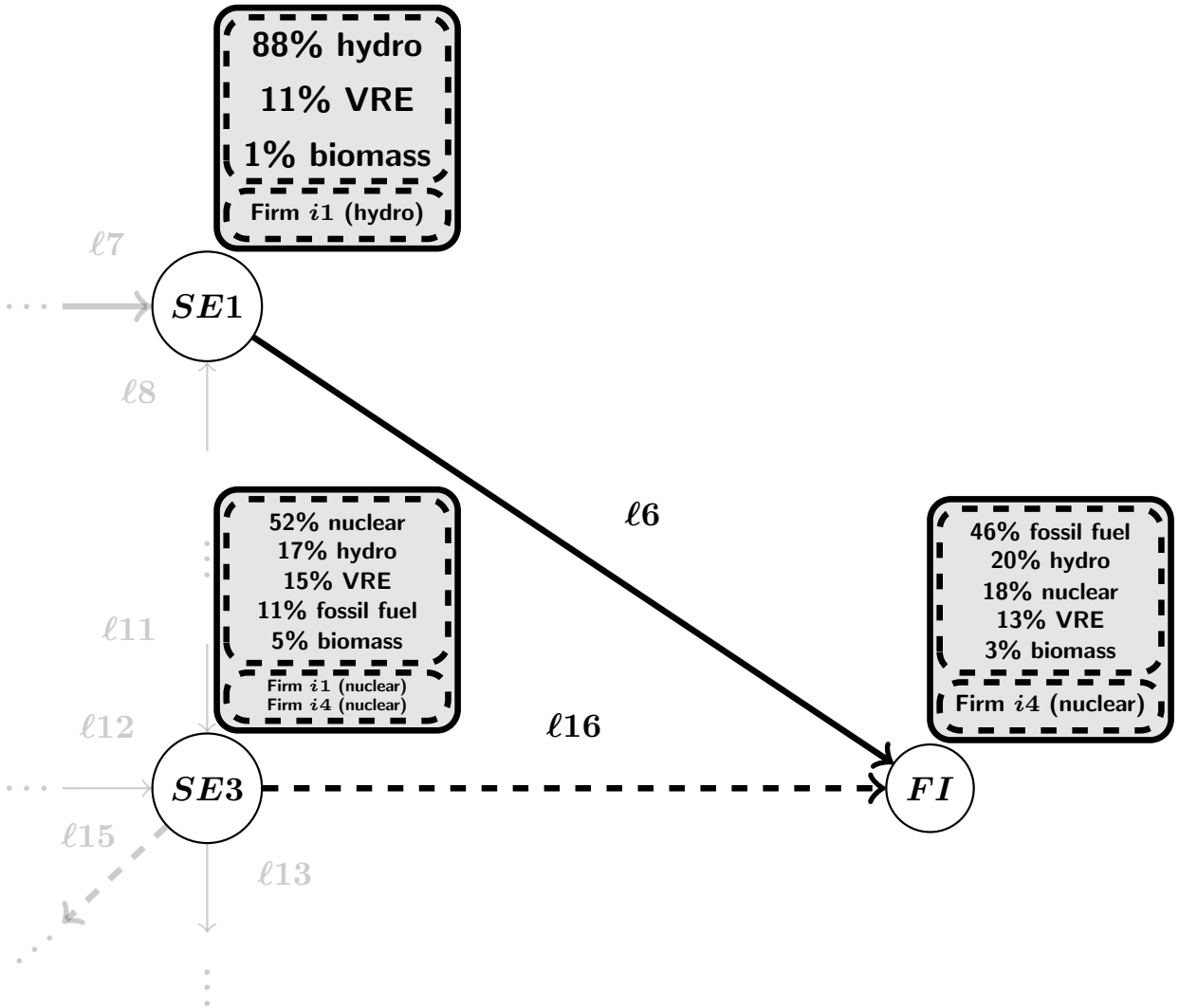

Summary Results in the FutureCVT Scenario (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.938	137.902	138.825
Consumer Surplus	129.658	125.177	128.780
Producer Surplus	8.397	11.052	9.128
Merchandising Surplus	0.947	1.837	0.980
Government Revenue	0.074	0.204	0.066
CO_2 Damage Cost	0.074	0.204	0.066
Transmission-Expansion Cost	0.064	0.163	0.064
CO_2 Emissions (Mt)	0.736	2.035	0.659
Firm $i1$'s Surplus	1.068	1.391	1.175
Average Price (€/MWh)	36.953	48.413	39.074
Generation Expansion (GW)	10.000	39.860	11.282
Transmission Expansion (–)	[0 0 0 2]	[2 0 0 2]	[0 0 0 2]

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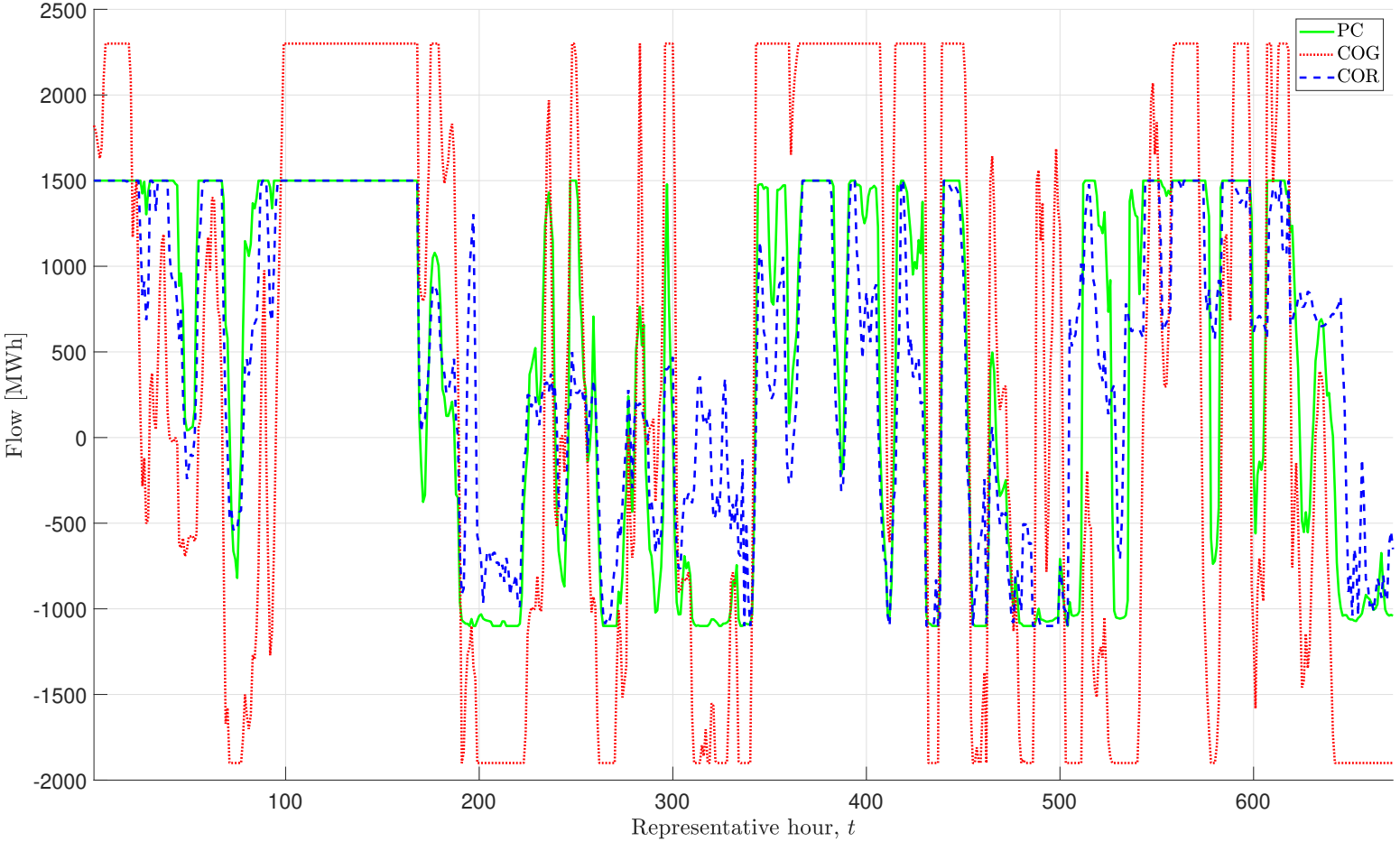
Hourly Flows on Line $\ell 6$ in the FutureCVT Scenario (in MWh)



Summary Results in the FutureCVT Scenario with $H = 0.15$ (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.365	136.197	138.150
Consumer Surplus	130.083	126.540	129.935
Producer Surplus	8.594	10.790	8.505
Merchandising Surplus	0.424	0.957	0.551
Government Revenue	0.116	0.355	0.131
CO_2 Damage Cost	0.771	2.364	0.870
Transmission-Expansion Cost	0.081	0.081	0.102
CO_2 Emissions (Mt)	7.713	23.639	8.704
Firm $i1$'s Surplus	1.162	1.320	1.123
Average Price (€/MWh)	37.368	46.457	37.288
Generation Expansion (GW)	7.824	34.309	8.638
Transmission Expansion (–)	[1 0 0 1]	[1 0 0 1]	[0 1 0 2]

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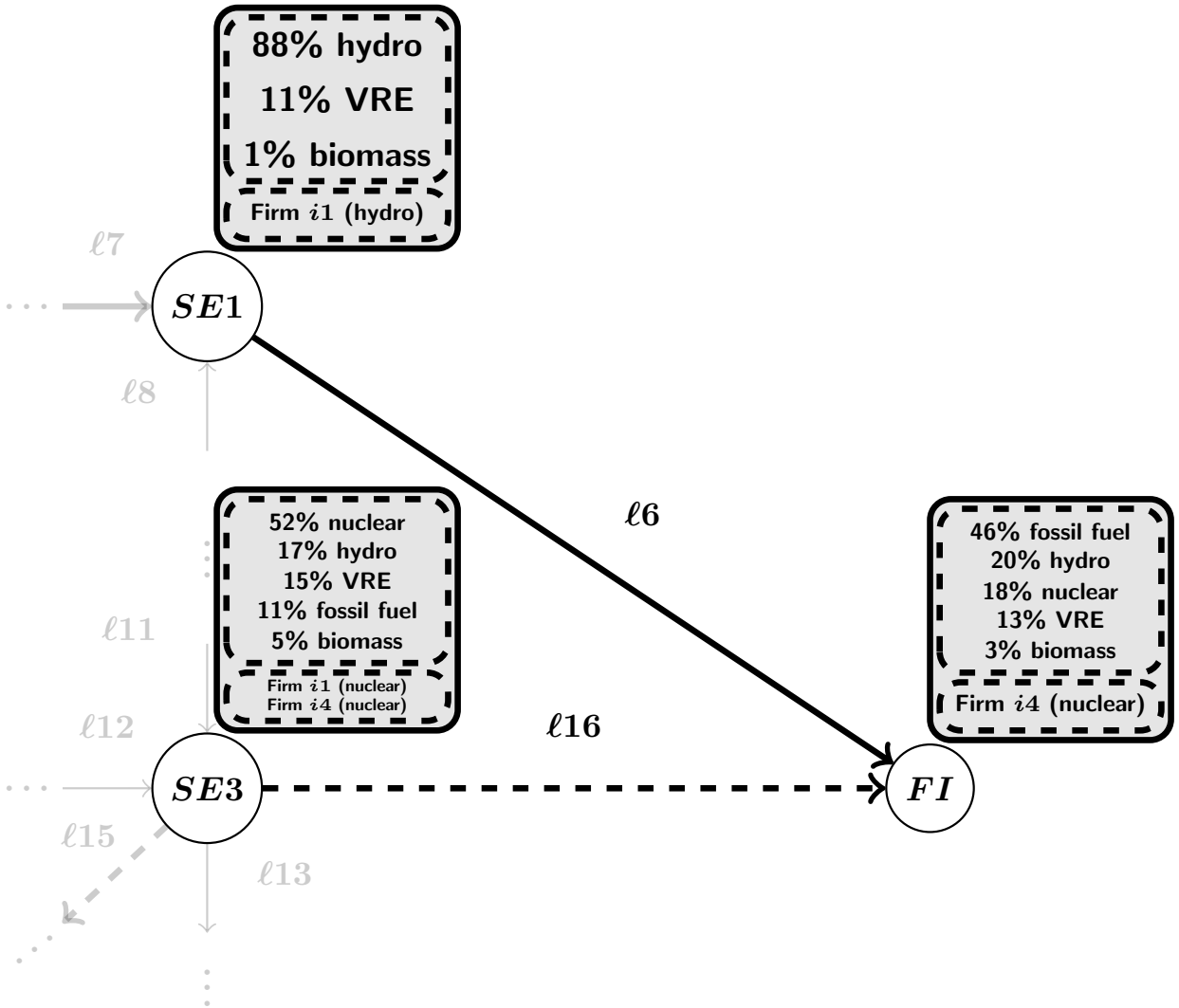
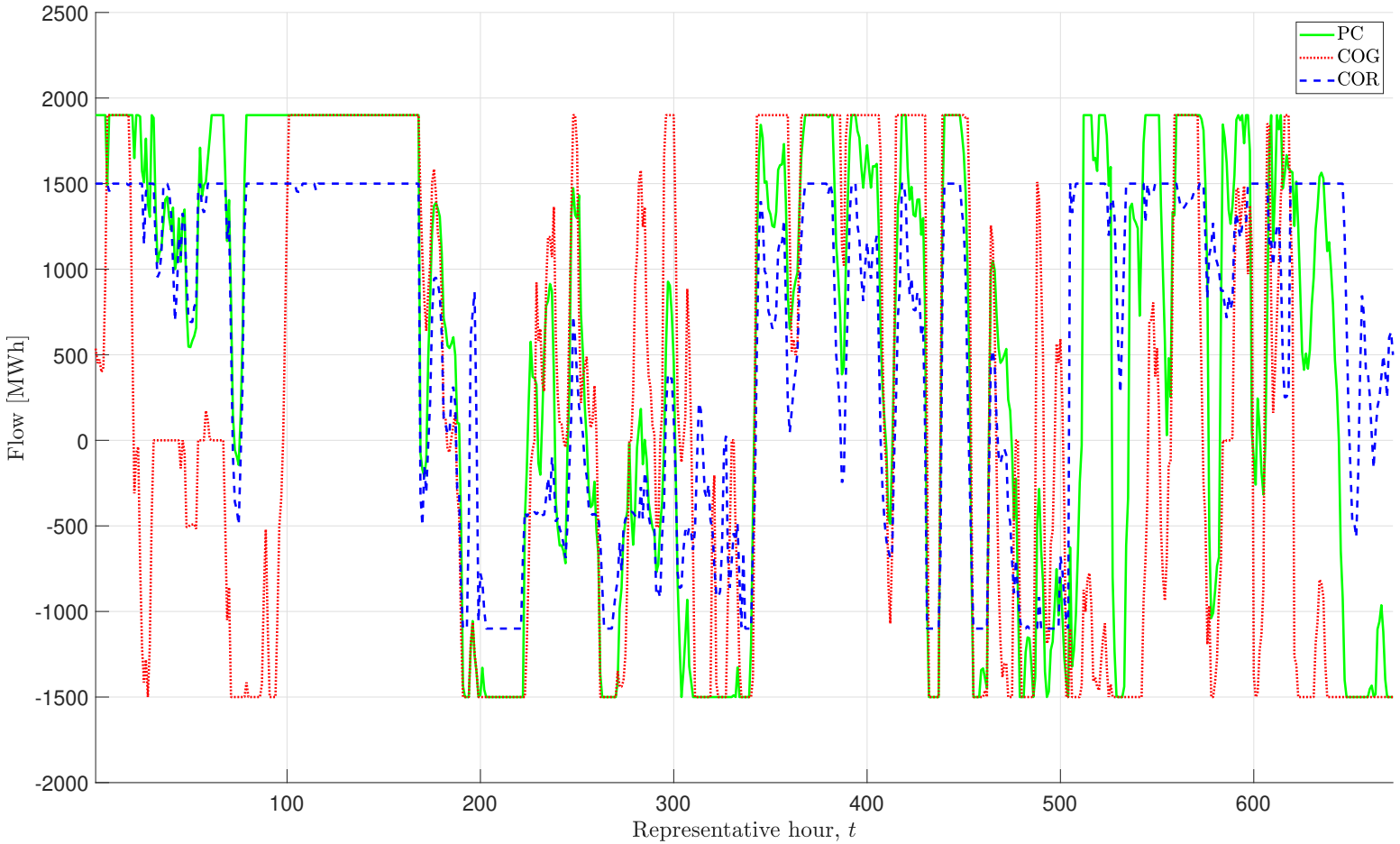
Network Map of Zones $SE1$, $SE3$, and FI with 2018 Generation-Capacity Mixes and Strategic Firms



Hourly Flows on Line $\ell 6$ in the FutureCVT Scenario with $H = 0.15$ (in MWh)



Conclusions

Summary

- Examine the implications of market imperfections and incomplete pricing of externalities for transmission expansion
  - A first-best transmission plan leverages full CO_2 pricing to curb consumption and to permit more efficient sharing of generation resources in facilitating VRE adoption (RQ 1)
  - Market power has contrasting impacts on transmission expansion (RQ 2): nuclear withholding necessitates additional transmission expansion to deploy hydro from $SE1$, while hydro arbitrage at $SE1$ entices VRE adoption at FI that utilises existing transmission capacity due to lower flows
  - Incomplete CO_2 pricing (RQ 3) means modifying the transmission plan under perfect competition to induce VRE adoption, while diluted scope for market power, i.e., less propensity to withhold (to conduct temporal arbitrage), means that less (more) transmission capacity is optimal
- Future work: regional trade, carbon leakage, endogenous carbon pricing, cooperative game among TSOs

Appendix

TSO's Problem

$$\underset{x_{j,\ell}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left[\left(D^{\text{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right)-\sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{U}_{i,n}}C_{i,n,t,u}g_{i,n,t,u}\right]$$
$$-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}}C^{\text{ava}}_{i,n,u}a_{i,n,u}-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}}C^{\text{ava}}_{e,i,n}a^{e}_{i,n}$$
$$-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}}C^{\text{ava}}_{i,n,w}a_{i,n,w}-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}}C^{\text{gen}}_{e,i,n}b^{e}_{i,n}$$
$$-\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}}SP_{i,n,u}g_{i,n,t,u}-\sum_{j\in\mathcal{J}_\ell}\sum_{\ell\in\mathcal{L}}C^{\text{trn}}_{j,\ell}x_{j,\ell} \quad (1)$$

$$\text{s.t. } x_{j,\ell}\in\{0,1\},\forall j\in\mathcal{J}_\ell,\ell \quad (2)$$

$$\sum_{j\in\mathcal{J}_\ell}x_{j,\ell}=1,\ \forall\ell \quad (3)$$

ISO's Problem

$$\underset{\boldsymbol{\Gamma^{\mathrm{ISO}}}}{\textbf{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\mathrm{int}}_{n,t}q_{n,t}-\frac{1}{2}D^{\mathrm{slp}}_{n,t}q^2_{n,t}\right) \tag{4}$$

$$\textbf{s.t. } q_{n,t}=\sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{U}_{i,n}}g_{i,n,t,u}+\sum_{i\in\mathcal{I}}\sum_{e\in\mathcal{E}_{i,n}}g^{e}_{i,n,t}+\sum_{i\in\mathcal{I}}\sum_{w\in\mathcal{W}_{i,n}}\Big(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Big)-\sum_{\ell\in\mathcal{L}^{+}_{n}}V\,T_t\hat{f}_{\ell,t}+\sum_{\ell\in\mathcal{L}^{-}_{n}}V\,T_t\hat{f}_{\ell,t}\ :\ \theta_{n,t},\forall n,t \tag{5}$$

$$\underline{\mu}_{j,\ell,t}\ :\ -T_t\underline{K}_{j,\ell}\le VT_tf_{j,\ell,t}\le T_t\overline{K}_{j,\ell}\ :\ \overline{\mu}_{j,\ell,t},\forall j\in\mathcal{J}_\ell,\ell,t \tag{6}$$

$$\hat{f}_{\ell,t}=\sum_{j\in\mathcal{J}_\ell}f_{j,\ell,t}\ :\ \psi_{\ell,t},\forall\ell,t \tag{7}$$

$$T_tf_{j,\ell^{\mathrm{AC}},t}=x_{j,\ell^{\mathrm{AC}}}T_tB_{j,\ell^{\mathrm{AC}}}\left(v_{n^{+}_{\ell},t}-v_{n^{-}_{\ell},t}\right)\ :\ \eta_{j,\ell^{\mathrm{AC}},t},$$

$$\forall j\in\mathcal{J}_\ell,\ell^{\mathrm{AC}}\in\mathcal{L}^{\mathrm{AC}},t \tag{8}$$

$$\underline{\kappa}_{n^{\mathrm{AC}},t}\ :\ -\pi\le v_{n^{\mathrm{AC}},t}\le\pi\ :\ \overline{\kappa}_{n^{\mathrm{AC}},t},\forall n^{\mathrm{AC}}\in\mathcal{N}^{\mathrm{AC}},t \tag{9}$$

where $\boldsymbol{\Gamma^{\mathrm{ISO}}}\equiv\{q_{n,t}\ge 0,\hat{f}_{\ell,t}\ \textbf{u.r.s.},f_{j,\ell,t}\ \textbf{u.r.s.},v_{n^{AC},t}\ \textbf{u.r.s.}\}$

ISO's Problem

$$\underset{\mathbf{\Gamma^{ISO}}}{\textbf{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(D^{\text{int}}_{n,t}q_{n,t} - \frac{1}{2}D^{\text{slp}}_{n,t}q^2_{n,t}\right) \tag{4}$$

$$\text{s.t. } q_{n,t} = \sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{i\in\mathcal{I}}\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t} + \sum_{i\in\mathcal{I}}\sum_{w\in\mathcal{W}_{i,n}}\Big(Q_{i,n,w}r^{\text{out}}_{i,n,t,w} - F_{i,n,w}r^{\text{in}}_{i,n,t,w}\Big) - \sum_{\ell\in\mathcal{L}^{+}_{n}} V\,T_t\hat{f}_{\ell,t} + \sum_{\ell\in\mathcal{L}^{-}_{n}} V\,T_t\hat{f}_{\ell,t} \;:\; \theta_{n,t}, \forall n,t \tag{5}$$

$$\underline{\mu}_{j,\ell,t} \;:\; -T_t\underline{K}_{j,\ell} \le VT_t f_{j,\ell,t} \le T_t\overline{K}_{j,\ell} \;:\; \overline{\mu}_{j,\ell,t}, \forall j\in\mathcal{J}_\ell, \ell, t \tag{6}$$

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where $\mathbf{\Gamma^{ISO}} \equiv \{q_{n,t}\ge 0, \hat{f}_{\ell,t}$ u.r.s., $f_{j,\ell,t}$ u.r.s., $v_{n^{AC},t}$ u.r.s.$\}$

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$$\underline{\kappa}_{n^{\text{AC}},t} : -\pi \le v_{n^{\text{AC}},t} \le \pi : \overline{\kappa}_{n^{\text{AC}},t}, \forall n^{\text{AC}}\in\mathcal{N}^{\text{AC}}, t \tag{9}$$

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Firm i's Problem

$$\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\Big(D^{\mathrm{int}}_{n,t}-D^{\mathrm{slp}}_{n,t}q_{n,t}\Big)\Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-\sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{ava}}_{i,n,u}a_{i,n,u}$$

$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\mathrm{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

$$\text{s.t. } g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\ \forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$

$$a_{i,n,u}\le \overline{G}_{i,n,u}\ :\ \beta^{\mathrm{ava}}_{i,n,u},\ \forall n,u\in\mathcal{U}_{i,n} \tag{12}$$

$$\beta^{\mathrm{down}}_{i,n,t,u}\ :\ -T_t R^{\mathrm{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\mathrm{up}}_{u}a_{i,n,u}\ :\ \beta^{\mathrm{up}}_{i,n,t,u},\ \forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$

$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\ \forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$

$$a^{e}_{i,n}\le \overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\mathrm{ava}}_{e,i,n},\ \forall e\in\mathcal{E}_{i,n},n \tag{15}$$

$$r^{\mathrm{sto}}_{i,n,t,w}=(1-E^{\mathrm{sto}}_{i,n,w})^{T_t}r^{\mathrm{sto}}_{i,n,t-1,w}+r^{\mathrm{in}}_{i,n,t,w}-r^{\mathrm{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\mathrm{bal}}_{i,n,t,w},$$

$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\mathrm{sto}}_{i,n,t,w}\le \overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$

$$r^{\mathrm{in}}_{i,n,t,w}\le T_t R^{\mathrm{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$

$$Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\mathrm{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$

$$a_{i,n,w}\le \overline{Y}_{i,n,w}\ :\ \beta^{\mathrm{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$

$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{\mathrm{i}}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\mathrm{in}}_{i,n,t,w}\ge 0, r^{\mathrm{out}}_{i,n,t,w}\ge 0, r^{\mathrm{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

Firm i's Problem

$$\underset{\Gamma^{i}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\Big(D^{\text{int}}_{n,t}-D^{\text{slp}}_{n,t}q_{n,t}\Big)\Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w}r^{\text{out}}_{i,n,t,w}$$
$$-\sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w}r^{\text{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{ava}}_{i,n,u}a_{i,n,u}$$
$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\text{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\text{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\text{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

$$\text{s.t. } g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$
$$a_{i,n,u}\le \overline{G}_{i,n,u}\ :\ \beta^{\text{ava}}_{i,n,u},\forall n,u\in\mathcal{U}_{i,n} \tag{12}$$
$$\beta^{\text{down}}_{i,n,t,u}\ :\ -T_t R^{\text{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\text{up}}_{u}a_{i,n,u}\ :\ \beta^{\text{up}}_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$
$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$
$$a^{e}_{i,n}\le \overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\text{ava}}_{e,i,n},\forall e\in\mathcal{E}_{i,n},n \tag{15}$$
$$r^{\text{sto}}_{i,n,t,w}=(1-E^{\text{sto}}_{i,n,w})^{T_t}r^{\text{sto}}_{i,n,t-1,w}+r^{\text{in}}_{i,n,t,w}-r^{\text{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\text{bal}}_{i,n,t,w},$$
$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$
$$\lambda^{\text{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\text{sto}}_{i,n,t,w}\le \overline{R}_{i,n,w}\ :\ \lambda^{\text{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$
$$r^{\text{in}}_{i,n,t,w}\le T_t R^{\text{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\text{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$
$$Q_{i,n,w}r^{\text{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\text{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$
$$a_{i,n,w}\le \overline{Y}_{i,n,w}\ :\ \beta^{\text{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$
$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\text{out}}_{i,n,t,w}-F_{i,n,w}r^{\text{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{i}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\text{in}}_{i,n,t,w}\ge 0, r^{\text{out}}_{i,n,t,w}\ge 0, r^{\text{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

Firm i's Problem

$$\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\Big(D^{\mathrm{int}}_{n,t}-D^{\mathrm{slp}}_{n,t}q_{n,t}\Big)\Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-\sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{ava}}_{i,n,u}a_{i,n,u}$$

$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\mathrm{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

s.t.

$$g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$

$$a_{i,n,u}\le \overline{G}_{i,n,u}\ :\ \beta^{\mathrm{ava}}_{i,n,u},\forall n,u\in\mathcal{U}_{i,n} \tag{12}$$

$$\beta^{\mathrm{down}}_{i,n,t,u}\ :\ -T_t R^{\mathrm{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\mathrm{up}}_{u}a_{i,n,u}\ :\ \beta^{\mathrm{up}}_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$

$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$

$$a^{e}_{i,n}\le \overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\mathrm{ava}}_{e,i,n},\forall e\in\mathcal{E}_{i,n},n \tag{15}$$

$$r^{\mathrm{sto}}_{i,n,t,w}=(1-E^{\mathrm{sto}}_{i,n,w})^{T_t}r^{\mathrm{sto}}_{i,n,t-1,w}+r^{\mathrm{in}}_{i,n,t,w}-r^{\mathrm{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\mathrm{bal}}_{i,n,t,w},$$

$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\mathrm{sto}}_{i,n,t,w}\le \overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$

$$r^{\mathrm{in}}_{i,n,t,w}\le T_t R^{\mathrm{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$

$$Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\mathrm{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$

$$a_{i,n,w}\le \overline{Y}_{i,n,w}\ :\ \beta^{\mathrm{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$

$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{\mathrm{i}}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\mathrm{in}}_{i,n,t,w}\ge 0, r^{\mathrm{out}}_{i,n,t,w}\ge 0, r^{\mathrm{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

Firm i's Problem

$$\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\left(D^{\mathrm{int}}_{n,t}-D^{\mathrm{slp}}_{n,t}q_{n,t}\right)\Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-\sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{ava}}_{i,n,u}a_{i,n,u}$$

$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\mathrm{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

$$\text{s.t. } g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$

$$a_{i,n,u}\le \overline{G}_{i,n,u}\ :\ \beta^{\mathrm{ava}}_{i,n,u},\forall n,u\in\mathcal{U}_{i,n} \tag{12}$$

$$\beta^{\mathrm{down}}_{i,n,t,u}\ :\ -T_t R^{\mathrm{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\mathrm{up}}_{u}a_{i,n,u}\ :\ \beta^{\mathrm{up}}_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$

$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$

$$a^{e}_{i,n}\le \overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\mathrm{ava}}_{e,i,n},\forall e\in\mathcal{E}_{i,n},n \tag{15}$$

$$r^{\mathrm{sto}}_{i,n,t,w}=(1-E^{\mathrm{sto}}_{i,n,w})^{T_t}r^{\mathrm{sto}}_{i,n,t-1,w}+r^{\mathrm{in}}_{i,n,t,w}-r^{\mathrm{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\mathrm{bal}}_{i,n,t,w},$$

$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\mathrm{sto}}_{i,n,t,w}\le \overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$

$$r^{\mathrm{in}}_{i,n,t,w}\le T_t R^{\mathrm{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$

$$Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\mathrm{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$

$$a_{i,n,w}\le \overline{Y}_{i,n,w}\ :\ \beta^{\mathrm{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$

$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{\mathrm{i}}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\mathrm{in}}_{i,n,t,w}\ge 0, r^{\mathrm{out}}_{i,n,t,w}\ge 0, r^{\mathrm{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

Firm i's Problem

$$\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\left(D^{\mathrm{int}}_{n,t}-D^{\mathrm{slp}}_{n,t}q_{n,t}\right)\Bigg(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}} g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}} Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-\sum_{w\in\mathcal{W}_{i,n}} F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{ava}}_{i,n,u}a_{i,n,u}$$

$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\mathrm{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

$$\text{s.t. } g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$

$$a_{i,n,u}\le \overline{G}_{i,n,u}\ :\ \beta^{\mathrm{ava}}_{i,n,u},\forall n,u\in\mathcal{U}_{i,n} \tag{12}$$

$$\beta^{\mathrm{down}}_{i,n,t,u}\ :\ -T_t R^{\mathrm{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\mathrm{up}}_{u}a_{i,n,u}\ :\ \beta^{\mathrm{up}}_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$

$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$

$$a^{e}_{i,n}\le \overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\mathrm{ava}}_{e,i,n},\forall e\in\mathcal{E}_{i,n},n \tag{15}$$

$$r^{\mathrm{sto}}_{i,n,t,w}=(1-E^{\mathrm{sto}}_{i,n,w})^{T_t}r^{\mathrm{sto}}_{i,n,t-1,w}+r^{\mathrm{in}}_{i,n,t,w}-r^{\mathrm{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\mathrm{bal}}_{i,n,t,w},$$

$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\mathrm{sto}}_{i,n,t,w}\le \overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$

$$r^{\mathrm{in}}_{i,n,t,w}\le T_t R^{\mathrm{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$

$$Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\mathrm{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$

$$a_{i,n,w}\le \overline{Y}_{i,n,w}\ :\ \beta^{\mathrm{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$

$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{\mathrm{i}}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\mathrm{in}}_{i,n,t,w}\ge 0, r^{\mathrm{out}}_{i,n,t,w}\ge 0, r^{\mathrm{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

Firm i's Problem

$$\underset{\Gamma^{\mathrm{i}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\Bigg[\left(D^{\mathrm{int}}_{n,t}-D^{\mathrm{slp}}_{n,t}q_{n,t}\right)\Bigg(\sum_{u\in\mathcal{U}_{i,n}}g_{i,n,t,u}+\sum_{e\in\mathcal{E}_{i,n}}g^{e}_{i,n,t}+\sum_{w\in\mathcal{W}_{i,n}}Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}$$

$$-\sum_{w\in\mathcal{W}_{i,n}}F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\Bigg)-\sum_{u\in\mathcal{U}_{i,n}}\left(C_{i,n,t,u}+HSP_{i,n,u}\right)g_{i,n,t,u}\Bigg]-\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}}C^{\mathrm{ava}}_{i,n,u}a_{i,n,u}$$

$$-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}}C^{\mathrm{ava}}_{e,i,n}a^{e}_{i,n}-\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}}C^{\mathrm{ava}}_{i,n,w}a_{i,n,w}-\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}}C^{\mathrm{gen}}_{e,i,n}b^{e}_{i,n} \tag{10}$$

$$\text{s.t. } g_{i,n,t,u}\le T_t a_{i,n,u}\ :\ \beta_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{11}$$

$$a_{i,n,u}\le\overline{G}_{i,n,u}\ :\ \beta^{\mathrm{ava}}_{i,n,u},\forall n,u\in\mathcal{U}_{i,n} \tag{12}$$

$$\beta^{\mathrm{down}}_{i,n,t,u}\ :\ -T_t R^{\mathrm{down}}_{u}a_{i,n,u}\le g_{i,n,t,u}-g_{i,n,t-1,u}\le T_t R^{\mathrm{up}}_{u}a_{i,n,u}\ :\ \beta^{\mathrm{up}}_{i,n,t,u},\forall n,t,u\in\mathcal{U}_{i,n} \tag{13}$$

$$g^{e}_{i,n,t}\le T_t A^{e}_{n,t}a^{e}_{i,n}\ :\ \beta^{e}_{i,n,t},\forall e\in\mathcal{E}_{i,n},n,t \tag{14}$$

$$a^{e}_{i,n}\le\overline{G}^{e}_{i,n}+b^{e}_{i,n}\ :\ \beta^{\mathrm{ava}}_{e,i,n},\forall e\in\mathcal{E}_{i,n},n \tag{15}$$

$$r^{\mathrm{sto}}_{i,n,t,w}=(1-E^{\mathrm{sto}}_{i,n,w})^{T_t}r^{\mathrm{sto}}_{i,n,t-1,w}+r^{\mathrm{in}}_{i,n,t,w}-r^{\mathrm{out}}_{i,n,t,w}-z_{i,n,t,w}+I_{i,n,t,w}\ :\ \lambda^{\mathrm{bal}}_{i,n,t,w},$$

$$\forall n,t,w\in\mathcal{W}_{i,n} \tag{16}$$

$$\lambda^{\mathrm{lb}}_{i,n,t,w}\ :\ \underline{R}_{i,n,w}\le r^{\mathrm{sto}}_{i,n,t,w}\le\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{ub}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{17}$$

$$r^{\mathrm{in}}_{i,n,t,w}\le T_t R^{\mathrm{in}}_{i,n,w}\overline{R}_{i,n,w}\ :\ \lambda^{\mathrm{in}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{18}$$

$$Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}\le T_t a_{i,n,w}\ :\ \lambda^{\mathrm{h}}_{i,n,t,w},\forall n,t,w\in\mathcal{W}_{i,n} \tag{19}$$

$$a_{i,n,w}\le\overline{Y}_{i,n,w}\ :\ \beta^{\mathrm{ava}}_{i,n,w},\forall n,w\in\mathcal{W}_{i,n} \tag{20}$$

$$\sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}_{i,n}}\left(Q_{i,n,w}r^{\mathrm{out}}_{i,n,t,w}-F_{i,n,w}r^{\mathrm{in}}_{i,n,t,w}\right)\ge Z_{i,n}\ :\ \gamma_{i,n},\forall n \tag{21}$$

where $\Gamma^{\mathrm{i}}\equiv\{a_{i,n,u}\ge 0, a^{e}_{i,n}\ge 0, a_{i,n,w}\ge 0, b^{e}_{i,n}\ge 0,\ g_{i,n,t,u}\ge 0, g^{e}_{i,n,t}\ge 0, r^{\mathrm{in}}_{i,n,t,w}\ge 0, r^{\mathrm{out}}_{i,n,t,w}\ge 0, r^{\mathrm{sto}}_{i,n,t,w}\ge 0, z_{i,n,t,w}\ge 0\}$

ISO's KKT Conditions

$$0 \leq q_{n,t} \perp -\left(D^{\text{int}}_{n,t} - D^{\text{slp}}_{n,t} q_{n,t}\right) + \theta_{n,t} \geq 0, \forall n, t \tag{22}$$

$$\hat{f}_{\ell,t} \text{ u.r.s., } \psi_{\ell,t} + VT_t \theta_{n^+_\ell,t} - VT_t \theta_{n^-_\ell,t} = 0, \forall \ell, t \tag{23}$$

$$f_{j,\ell,t} \text{ u.r.s., } T_t \eta_{j,\ell^{\text{AC}},t} + V\ T_t \overline{\mu}_{j,\ell,t} - V\ T_t \underline{\mu}_{j,\ell,t} - \psi_{\ell,t} = 0, \forall j \in \mathcal{J}_\ell, \ell, t \tag{24}$$

$$v_{n^{\text{AC}},t} \text{ u.r.s., } -\sum_{\ell \in \mathcal{L}^+_n} \sum_{j \in \mathcal{J}_\ell} x_{j,\ell^{\text{AC}}} T_t B_{j,\ell^{\text{AC}}} \eta_{\ell^{\text{AC}},t} + \sum_{\ell \in \mathcal{L}^-_n} \sum_{j \in \mathcal{J}_\ell} x_{j,\ell^{\text{AC}}} T_t B_{j,\ell^{\text{AC}}} \eta_{\ell^{\text{AC}},t}$$
$$+ \overline{\kappa}_{n^{\text{AC}},t} - \underline{\kappa}_{n^{\text{AC}},t} = 0, \forall n^{\text{AC}} \in \mathcal{N}^{\text{AC}}, t \tag{25}$$

$$\theta_{n,t} \text{ u.r.s., } q_{n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} g_{i,n,t,u} - \sum_{i \in \mathcal{I}} \sum_{e \in \mathcal{E}_{i,n}} g^e_{i,n,t} - \sum_{i \in \mathcal{I}} \sum_{w \in \mathcal{W}_{i,n}} \Big(Q_{i,n,w} r^{\text{out}}_{i,n,t,w}$$
$$- F_{i,n,w} r^{\text{in}}_{i,n,t,w}\Big) + \sum_{\ell \in \mathcal{L}^+_n} V\ T_t \hat{f}_{\ell,t} - \sum_{\ell \in \mathcal{L}^-_n} V\ T_t \hat{f}_{\ell,t} = 0, \forall n, t \tag{26}$$

$$0 \leq \underline{\mu}_{j,\ell,t} \perp T_t \underline{K}_{j,\ell} + VT_t f_{j,\ell,t} \geq 0, \forall j \in \mathcal{J}_\ell, \ell, t \tag{27}$$

$$0 \leq \overline{\mu}_{j,\ell,t} \perp T_t \overline{K}_{j,\ell} - VT_t f_{j,\ell,t} \geq 0, \forall j \in \mathcal{J}_\ell, \ell, t \tag{28}$$

$$\eta_{j,\ell^{\text{AC}},t} \text{ u.r.s., } x_{j,\ell^{\text{AC}}} T_t B_{j,\ell^{\text{AC}}} \left(v_{n^+_\ell,t} - v_{n^+_\ell,t}\right) - T_t f_{j,\ell^{\text{AC}},t} = 0, \forall j \in \mathcal{J}_\ell, \ell^{\text{AC}} \in \mathcal{L}^{\text{AC}}, t \tag{29}$$

$$\psi_{\ell,t} \text{ u.r.s., } \hat{f}_{\ell,t} - \sum_{j \in \mathcal{J}_\ell} f_{j,\ell,t} = 0, \forall \ell, t \tag{30}$$

$$0 \leq \underline{\kappa}_{n^{\text{AC}},t} \perp \pi + v_{n^{\text{AC}},t} \geq 0, \forall n^{\text{AC}} \in \mathcal{N}^{\text{AC}}, t \tag{31}$$

$$0 \leq \overline{\kappa}_{n^{\text{AC}},t} \perp \pi - v_{n^{\text{AC}},t} \geq 0, \forall n^{\text{AC}} \in \mathcal{N}^{\text{AC}}, t \tag{32}$$

Firm i's KKT Conditions

$$0 \leq \boldsymbol{a}_{i,n,u} \perp \boldsymbol{C}^{\text{ava}}_{i,n,u} + \boldsymbol{\beta}^{\text{ava}}_{i,n,u} - \sum_{t\in\mathcal{T}} \boldsymbol{T}_t \boldsymbol{\beta}_{i,n,t,u} - \sum_{t\in\mathcal{T}} \boldsymbol{T}_t \boldsymbol{R}^{\text{up}}_u \boldsymbol{\beta}^{\text{up}}_{i,n,t,u} - \sum_{t\in\mathcal{T}} \boldsymbol{T}_t \boldsymbol{R}^{\text{down}}_u \boldsymbol{\beta}^{\text{down}}_{i,n,t,u} \geq \boldsymbol{0},\ \forall \boldsymbol{n}, \boldsymbol{u} \in \boldsymbol{\mathcal{U}}_{i,n} \tag{33}$$

$$0 \leq \boldsymbol{a}^{e}_{i,n} \perp \boldsymbol{C}^{\text{ava}}_{e,i,n} + \boldsymbol{\beta}^{\text{ava}}_{e,i,n} - \sum_{t\in\mathcal{T}} \boldsymbol{T}_t \boldsymbol{A}^{e}_{n,t} \boldsymbol{\beta}^{e}_{i,n,t} \geq \boldsymbol{0},\ \forall \boldsymbol{e} \in \boldsymbol{\mathcal{E}}_{i,n}, \boldsymbol{n} \tag{34}$$

$$0 \leq \boldsymbol{a}_{i,n,w} \perp \boldsymbol{C}^{\text{ava}}_{i,n,w} + \boldsymbol{\beta}^{\text{ava}}_{i,n,w} - \sum_{t\in\mathcal{T}} \boldsymbol{T}_t \boldsymbol{\lambda}^{\text{h}}_{i,n,t,w} \geq \boldsymbol{0},\ \forall \boldsymbol{n}, \boldsymbol{w} \in \boldsymbol{\mathcal{W}}_{i,n} \tag{35}$$

$$0 \leq \boldsymbol{b}^{e}_{i,n} \perp \boldsymbol{C}^{\text{gen}}_{e,i,n} - \boldsymbol{\beta}^{\text{ava}}_{e,i,n} \geq \boldsymbol{0},\ \forall \boldsymbol{e} \in \boldsymbol{\mathcal{E}}_{i,n}, \boldsymbol{n} \tag{36}$$

$$\begin{aligned} 0 \leq \boldsymbol{g}_{i,n,t,u} \perp \Big[& \boldsymbol{C}_{i,n,t,u} + \boldsymbol{HSP}_{i,n,u} - \left(\boldsymbol{D}^{\text{int}}_{n,t} - \boldsymbol{D}^{\text{slp}}_{n,t} \boldsymbol{q}_{n,t}\right) \\ & + \boldsymbol{D}^{\text{slp}}_{n,t} \left(\sum_{u'\in\mathcal{U}_{i,n}} \boldsymbol{g}_{i,n,t,u'} + \sum_{w\in\mathcal{W}_{i,n}} \boldsymbol{Q}_{i,n,w} \boldsymbol{r}^{\text{out}}_{i,n,t,w} - \sum_{w\in\mathcal{W}_{i,n}} \boldsymbol{F}_{i,n,w} \boldsymbol{r}^{\text{in}}_{i,n,t,w} \right) \Big] \\ & + \boldsymbol{\beta}_{i,n,t,u} + \boldsymbol{\beta}^{\text{up}}_{i,n,t,u} - \boldsymbol{\beta}^{\text{up}}_{i,n,t+1,u} + \boldsymbol{\beta}^{\text{down}}_{i,n,t+1,u} - \boldsymbol{\beta}^{\text{down}}_{i,n,t,u} \geq \boldsymbol{0}, \forall \boldsymbol{n}, \boldsymbol{t}, \boldsymbol{u} \in \boldsymbol{\mathcal{U}}_{i,n} \end{aligned} \tag{37}$$

$$0 \leq \boldsymbol{g}^{e}_{i,n,t} \perp -\left(\boldsymbol{D}^{\text{int}}_{n,t} - \boldsymbol{D}^{\text{slp}}_{n,t} \boldsymbol{q}_{n,t}\right) + \boldsymbol{\beta}^{e}_{i,n,t} \geq \boldsymbol{0},\ \forall \boldsymbol{e} \in \boldsymbol{\mathcal{E}}_{i,n}, \boldsymbol{n}, \boldsymbol{t} \tag{38}$$

$$0 \leq \boldsymbol{r}^{\text{sto}}_{i,n,t,w} \perp \boldsymbol{\lambda}^{\text{bal}}_{i,n,t,w} - (\boldsymbol{1} - \boldsymbol{E}^{\text{sto}}_{i,n,w})^{T_t} \boldsymbol{\lambda}^{\text{bal}}_{i,n,t+1,w} + \boldsymbol{\lambda}^{\text{ub}}_{i,n,t,w} - \boldsymbol{\lambda}^{\text{lb}}_{i,n,t,w} \geq \boldsymbol{0}, \forall \boldsymbol{n}, \boldsymbol{t}, \boldsymbol{w} \in \boldsymbol{\mathcal{W}}_{i,n} \tag{39}$$

$$0 \leq \boldsymbol{z}_{i,n,t,w} \perp \boldsymbol{\lambda}^{\text{bal}}_{i,n,t,w} \geq \boldsymbol{0}, \forall \boldsymbol{n}, \boldsymbol{t}, \boldsymbol{w} \in \boldsymbol{\mathcal{W}}_{i,n} \tag{40}$$

$$\begin{aligned} 0 \leq \boldsymbol{r}^{\text{in}}_{i,n,t,w} \perp \Big[& \boldsymbol{F}_{i,n,w} \left(\boldsymbol{D}^{\text{int}}_{n,t} - \boldsymbol{D}^{\text{slp}}_{n,t} \boldsymbol{q}_{n,t}\right) \\ & - \boldsymbol{F}_{i,n,w} \boldsymbol{D}^{\text{slp}}_{n,t} \left(\sum_{u\in\mathcal{U}_{i,n}} \boldsymbol{g}_{i,n,t,u} + \sum_{w'\in\mathcal{W}_{i,n}} \boldsymbol{Q}_{i,n,w'} \boldsymbol{r}^{\text{out}}_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} \boldsymbol{F}_{i,n,w'} \boldsymbol{r}^{\text{in}}_{i,n,t,w'} \right) \Big] \\ & - \boldsymbol{\lambda}^{\text{bal}}_{i,n,t,w} + \boldsymbol{\lambda}^{\text{in}}_{i,n,t,w} + \boldsymbol{F}_{i,n,w} \boldsymbol{\gamma}_{i,n} \geq \boldsymbol{0}, \forall \boldsymbol{n}, \boldsymbol{t}, \boldsymbol{w} \in \boldsymbol{\mathcal{W}}_{i,n} \end{aligned} \tag{41}$$

$$\begin{aligned} 0 \leq \boldsymbol{r}^{\text{out}}_{i,n,t,w} \perp \Big[& -\boldsymbol{Q}_{i,n,w} \left(\boldsymbol{D}^{\text{int}}_{n,t} - \boldsymbol{D}^{\text{slp}}_{n,t} \boldsymbol{q}_{n,t}\right) \\ & + \boldsymbol{Q}_{i,n,w} \boldsymbol{D}^{\text{slp}}_{n,t} \left(\sum_{u\in\mathcal{U}_{i,n}} \boldsymbol{g}_{i,n,t,u} + \sum_{w'\in\mathcal{W}_{i,n}} \boldsymbol{Q}_{i,n,w'} \boldsymbol{r}^{\text{out}}_{i,n,t,w'} - \sum_{w'\in\mathcal{W}_{i,n}} \boldsymbol{F}_{i,n,w'} \boldsymbol{r}^{\text{in}}_{i,n,t,w'} \right) \Big] \\ & + \boldsymbol{\lambda}^{\text{bal}}_{i,n,t,w} + \boldsymbol{Q}_{i,n,w} \boldsymbol{\lambda}^{\text{h}}_{i,n,t,w} - \boldsymbol{Q}_{i,n,w} \boldsymbol{\gamma}_{i,n} \geq \boldsymbol{0}, \forall \boldsymbol{n}, \boldsymbol{t}, \boldsymbol{w} \in \boldsymbol{\mathcal{W}}_{i,n} \end{aligned} \tag{42}$$

Firm i's KKT Conditions (cont'd)

$$\boldsymbol{\lambda}^{\text{bal}}_{i,n,t,w} \text{ u.r.s., } \boldsymbol{r}^{\text{sto}}_{i,n,t,w} - (1 - \boldsymbol{E}^{\text{sto}}_{i,n,w})^{T_t} \boldsymbol{r}^{\text{sto}}_{i,n,t-1,w} - \boldsymbol{r}^{\text{in}}_{i,n,t,w} + \boldsymbol{r}^{\text{out}}_{i,n,t,w} + \boldsymbol{z}_{i,n,t,w} - \boldsymbol{I}_{i,n,t,w} = 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{43}$$

$$0 \leq \boldsymbol{\beta}^{e}_{i,n,t} \perp \boldsymbol{T}_t \boldsymbol{A}^{e}_{n,t} \boldsymbol{a}^{e}_{i,n} - \boldsymbol{g}^{e}_{i,n,t} \geq 0, \forall e \in \mathcal{E}_{i,n}, n, t \tag{44}$$

$$0 \leq \boldsymbol{\beta}^{\text{ava}}_{e,i,n} \perp \overline{\boldsymbol{G}}^{e}_{i,n} + \boldsymbol{b}^{e}_{i,n} - \boldsymbol{a}^{e}_{i,n} \geq 0, \forall e \in \mathcal{E}_{i,n}, n \tag{45}$$

$$0 \leq \boldsymbol{\beta}_{i,n,t,u} \perp \boldsymbol{T}_t \boldsymbol{a}_{i,n,u} - \boldsymbol{g}_{i,n,t,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{46}$$

$$0 \leq \boldsymbol{\beta}^{\text{ava}}_{i,n,u} \perp \overline{\boldsymbol{G}}_{i,n,u} - \boldsymbol{a}_{i,n,u} \geq 0, \forall n, u \in \mathcal{U}_{i,n} \tag{47}$$

$$0 \leq \boldsymbol{\beta}^{\text{up}}_{i,n,t,u} \perp \boldsymbol{T}_t \boldsymbol{R}^{\text{up}}_{u} \boldsymbol{a}_{i,n,u} + \boldsymbol{g}_{i,n,t-1,u} - \boldsymbol{g}_{i,n,t,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{48}$$

$$0 \leq \boldsymbol{\beta}^{\text{down}}_{i,n,t,u} \perp \boldsymbol{T}_t \boldsymbol{R}^{\text{down}}_{u} \boldsymbol{a}_{i,n,u} + \boldsymbol{g}_{i,n,t,u} - \boldsymbol{g}_{i,n,t-1,u} \geq 0, \forall n, t, u \in \mathcal{U}_{i,n} \tag{49}$$

$$0 \leq \boldsymbol{\lambda}^{\text{in}}_{i,n,t,w} \perp \boldsymbol{T}_t \boldsymbol{R}^{\text{in}}_{i,n,w} \overline{\boldsymbol{R}}_{i,n,w} - \boldsymbol{r}^{\text{in}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{50}$$

$$0 \leq \boldsymbol{\lambda}^{\text{h}}_{i,n,t,w} \perp \boldsymbol{T}_t \boldsymbol{a}_{i,n,w} - \boldsymbol{Q}_{i,n,w} \boldsymbol{r}^{\text{out}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{51}$$

$$0 \leq \boldsymbol{\beta}^{\text{ava}}_{i,n,w} \perp \overline{\boldsymbol{Y}}_{i,n,w} - \boldsymbol{a}_{i,n,w} \geq 0, \forall n, w \in \mathcal{W}_{i,n} \tag{52}$$

$$0 \leq \boldsymbol{\lambda}^{\text{ub}}_{i,n,t,w} \perp \overline{\boldsymbol{R}}_{i,n,w} - \boldsymbol{r}^{\text{sto}}_{i,n,t,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{53}$$

$$0 \leq \boldsymbol{\lambda}^{\text{lb}}_{i,n,t,w} \perp \boldsymbol{r}^{\text{sto}}_{i,n,t,w} - \underline{\boldsymbol{R}}_{i,n,w} \geq 0, \forall n, t, w \in \mathcal{W}_{i,n} \tag{54}$$

$$0 \leq \boldsymbol{\gamma}_{i,n} \perp \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}_{i,n}} \left(\boldsymbol{Q}_{i,n,w} \boldsymbol{r}^{\text{out}}_{i,n,t,w} - \boldsymbol{F}_{i,n,w} \boldsymbol{r}^{\text{in}}_{i,n,t,w} \right) - \boldsymbol{Z}_{i,n} \geq 0, \forall n \tag{55}$$

Equivalent Lower-Level QP Problem (Hashimoto, 1985)

$$
\begin{aligned}
\underset{\Omega^{\mathrm{LL}}}{\text{Maximise}} \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \Bigg[& \left(D^{\mathrm{int}}_{n,t} q_{n,t} - \frac{1}{2} D^{\mathrm{slp}}_{n,t} q^2_{n,t} \right) \\
& - \sum_{i\in\mathcal{I}} \left\{ \frac{D^{\mathrm{slp}}_{n,t}}{2} \left(\sum_{u\in\mathcal{U}_{i,n}} g_{i,n,t,u} + \sum_{w\in\mathcal{W}_{i,n}} \left(Q_{i,n,w} r^{\mathrm{out}}_{i,n,t,w} - F_{i,n,w} r^{\mathrm{in}}_{i,n,t,w} \right) \right)^2 \right. \\
& \left. + \sum_{u\in\mathcal{U}_{i,n}} \left(C_{i,n,t,u} + HSP_{i,n,u} \right) g_{i,n,t,u} \right\} \Bigg] \\
& - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{ava}}_{i,n,u} a_{i,n,u} - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{ava}}_{e,i,n} a^e_{i,n} \\
& - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}_{i,n}} C^{\mathrm{ava}}_{i,n,w} a_{i,n,w} - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}_{i,n}} C^{\mathrm{gen}}_{e,i,n} b^e_{i,n} \qquad (56)
\end{aligned}
$$

s.t. (5) – (9)

(11) – (21), $\forall i \in \mathcal{I}$

where Ω^{LL} comprises the ISO's decisions, Γ^{ISO}, and all of the firms' decisions, Γ^{i}, $\forall i \in \mathcal{I}$

Bi-Level Formulation

$$\begin{aligned}
&\underset{\{x_{j,\ell}\}\cup\Omega^{\mathrm{LL}}}{\textbf{Maximise}} && (1)\\
&\textbf{s.t.} && (2)-(3)\\
&&& \underset{\Omega^{\mathrm{LL}}}{\textbf{Maximise}} \quad (56)\\
&&& \textbf{s.t.} \quad (5)-(9)\\
&&& \qquad (11)-(21),\ \forall i \in \mathcal{I}
\end{aligned}$$

Summary Results in the FutureC Scenario (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.057	135.706	137.929
Consumer Surplus	117.355	102.567	116.808
Producer Surplus	19.255	32.062	19.512
Merchandising Surplus	1.448	1.077	1.609
Government Revenue	0.522	1.469	0.514
CO_2 Damage Cost	0.522	1.469	0.514
CO_2 Emissions (Mt)	5.224	14.690	5.139
Firm $i1$'s Surplus	3.064	4.086	3.071
Average Price (€/MWh)	64.753	108.991	65.516

Summary Results in the FutureCV Scenario (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.913	137.829	138.797
Consumer Surplus	129.291	124.742	128.176
Producer Surplus	8.550	11.238	9.533
Merchandising Surplus	1.073	1.849	1.088
Government Revenue	0.105	0.342	0.096
CO_2 Damage Cost	0.105	0.342	0.096
CO_2 Emissions (Mt)	1.053	3.420	0.958
Firm $i1$'s Surplus	1.049	1.372	1.190
Average Price (€/MWh)	37.151	48.746	39.933
Generation Expansion (GW)	9.916	39.190	11.035

Summary Results in the FutureCV Scenario with $H = 0.15$ (in Billion € Unless Indicated)

Metric \ Case	PC	COG	COR
Social Welfare	138.315	136.137	138.106
Consumer Surplus	130.120	126.511	129.677
Producer Surplus	8.488	10.774	8.718
Merchandising Surplus	0.474	0.952	0.567
Government Revenue	0.135	0.370	0.151
CO_2 Damage Cost	0.903	2.471	1.007
CO_2 Emissions (Mt)	9.027	24.710	10.075
Firm $i1$'s Surplus	1.133	1.319	1.139
Average Price (€/MWh)	37.033	46.408	37.743
Generation Expansion (GW)	7.400	34.051	8.196