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Because they cannot find a robust solution without constantly changing their minds.

What can we do to help?

With smart algorithms we can support two-stage/adjustable robust problems to find a robust optimal solution!

This joke was created with the help of ChatGPT

Adjustable Robust Network Design for Energy Networks

Johannes Thürauf, Julia Grübel, Martin Schmidt

Variational Analysis and Applications for Modeling of Energy Exchange (VAME)
May 13, 2024

Adjustable Robust Nonlinear Network Design

Task

Compute a network design taking into account demand uncertainties

Consider an accurate nonconvex transport model

$\rightarrow$ Adjustable robust MINLP

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Discrete decisions and nonlinear constraints

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Key Components of the Solution Approach

Exploit the underlying network and structural properties of potential-based flows

Overview

Potential-Based Flows

Robust Network Design Model

Characterizing Worst-Case Scenarios

Computational Results

Potential-Based Flows

Potential-Based Flows

Network modeled as a digraph $G = (V, A)$ with $V := V_+ \cup V_- \cup V_0$

Balanced load flow $\ell \in \mathbb{R}^V$, i.e., $\sum_{u \in V_+} \ell_u = \sum_{u \in V_-} \ell_u$, is feasible if $\exists\, q, \pi$ with

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$$\sum_{a \in \delta^{\text{out}}(v)} q_a - \sum_{a \in \delta^{\text{in}}(v)} q_a = \begin{cases} \ell_v, & \text{if } v \in V_+ \\ -\ell_v, & \text{if } v \in V_-, \\ 0, & \text{else} \end{cases} \quad v \in V$$

$$q_a^- \le q_a \le q_a^+, \quad a \in A$$

$$\pi_u - \pi_v = \Lambda_a \varphi(q_a), \quad a = (u, v) \in A$$

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We consider potential functions of the form $\varphi(q_a) = q_a |q_a|^r$ with $r \geq 0$

$\rightarrow$ allows to model gas, hydrogen, water, and lossless DC power flow networks

Robust Network Design Model

Network Expansion

Expansion variables $x_a \in \{0, 1\}$ for $a \in A_{\text{ca}}$

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$$(1 - x_a) M^- \le \pi_u - \pi_v - \Lambda_a \varphi(q_a) \le (1 - x_a) M^+, \quad a \in A_{\text{ca}}$$

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Nominal Network Design: Model

Mixed-integer nonconvex optimization problem

$$\begin{aligned}
\min_{x,q,\pi} \quad & \sum_{a \in A_{\text{ca}}} c_a x_a \\
\text{s.t.} \quad & x \in X \subseteq \{0,1\}^{A_{\text{ca}}} \\
& \text{massflow conservation}(q;\ell), \quad u \in V \\
& \text{potential-based flows}(q,\pi), \quad a \in A \\
& \text{potential-based flows expansion}(q,\pi), \quad a \in A_{\text{ca}} \\
& \text{potential and flow bounds}(q,\pi), \quad u \in V,\ a \in A
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Demand fluctuations can lead to infeasibility of the computed network design!

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Demand fluctuations can lead to infeasibility of the computed network design!
$\rightarrow$ consider demand uncertainties

Modeling Demand Uncertainty

Robust optimization approach
$\rightarrow$ Protect against all demand fluctuations within the uncertainty set

$$U := \left\{ \ell \in \mathbb{R}_{\geq 0} : \sum_{u \in V_+} \ell_u = \sum_{u \in V_-} \ell_u,\ \ell_u = 0\ \forall u \in V_0 \right\} \cap Z$$

with Z being a compact set

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General form of the uncertainty set: polyhedral, ellipsoidal, ...
$\rightarrow$ covers different situations of demand uncertainties

Adjustable robust nonconvex optimization problem:

$$
\begin{aligned}
\min_{x,q,\pi} \quad & \sum_{a \in A_{\text{ca}}} c_a x_a \\
\text{s.t.} \quad & x \in X \subseteq \{0,1\}^{A_{\text{ca}}} \\
& \forall \ell \in U \; \exists \, q, \pi \text{ that satisfy} \\
& \qquad \text{massflow conservation}(q_\ell; \ell), \quad u \in V \\
& \qquad \text{potential-based flows}(q_\ell, \pi_\ell), \quad a \in A_{\text{ca}} \\
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How can we solve this challenging problem?

Adversarial Solution Approach

General Algorithmic Idea

Determine a set of finitely many scenarios $S \subseteq U$

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Well known approach in strictly robust optimization; see e.g., Yanıkoğlu et al. 2019

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How can we find violating scenarios for the adjustable robust nonconvex problem?

Can we guarantee finite termination?

Characterizing Worst-Case Scenarios

Finding Worst-Case Scenarios

Three types of “worst-case” scenarios

- Unbalanced demands between different connected components
- Violating flow bounds
- Violating potential bounds

Worst-Case Scenarios: Unbalanced Demands

Fixed network expansion $x \in X$ and the expanded graph $G(x) = (V, A(x))$

Connected component $G^i = (V^i, A^i)$

Worst-Case Scenarios: Unbalanced Demands

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Find unbalanced demands

$$\mu_{G^i}(x) := \max_{\ell} |y| \quad \text{s.t.} \quad y = \sum_{u \in V^i \cap V_+} \ell_u - \sum_{u \in V^i \cap V_-} \ell_u,\ \ell \in U$$

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$\mu_{G^i}(x) > 0 \rightarrow x$ is robust infeasible

$\rightarrow$ At most $|V|$ many worst-case scenarios

Worst-Case Scenarios: Flow Bounds

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Minimum arc flow in U

$$\underline{q}_a(x) := \min_{\ell, q, \pi} \quad q_a \quad \text{s.t.} \quad \begin{aligned} &\text{massflow conservation}, && u \in V \\ &\text{potential-based flows}, && a = (u, v) \in A \\ &\ell \in U, && \text{no bounds} \end{aligned}$$

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$\rightarrow$ At most $2|A(x)|$ many worst-case flow scenarios for fixed x

Worst-Case Scenarios: Potential Bounds

Maximum potential difference between pair (u, v) of nodes

$$\varphi_{u,v}(x) := \max_{\ell,q,\pi} \quad \pi_u - \pi_v \quad \text{s.t.} \quad \begin{aligned} &\text{massflow conservation}, \quad u \in V \\ &\text{potential-based flows}, \quad a = (u, v) \in A \\ &\ell \in U, \quad \text{no bounds} \end{aligned}$$

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→ At most $|V|^2$ many worst-case scenarios for the potential bounds

Main Result: Characterization of Robust Feasibility

Theorem

Let $x \in X$ be fixed and $G'(x) = (V, A_{ex} \cup \{a \in A_{ca} \colon x_a = 1\})$ be the expanded graph. Let $\mathcal{G}'(x) := \{G^1, \dots, G^n\}$ with $G^i = (V^i, A^i)$ be the set of connected components of $G'(x)$. Then, expansion x is adjustable robust feasible if and only if

$$
\begin{aligned}
\mu_{G^i}(x) &= 0 && \text{for all} \quad G^i \in \mathcal{G}'(x) \\
\varphi_{u,v}(x) &\leq \pi_u^+ - \pi_v^- && \text{for all} \quad (u,v) \in (V^i)^2,\ G^i \in \mathcal{G}'(x) \\
\underline{q}_a(x) &\geq q_a^- && \text{for all} \quad a \in A^i,\ G^i \in \mathcal{G}'(x) \\
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$\rightarrow$ At most $|V| + |V|^2 + 2|A|$ many “worst-case” scenarios

Result holds for general compact uncertainty sets U

General Algorithmic Idea

Determine a set of finitely many scenarios $S \subseteq U$

Solve robust network design problem w.r.t. S instead of $U \leftarrow \; (x, q, \pi)$

Compute the finitely many "worst-case" scenarios w.r.t. fixed x

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Variant: Add at most one violating scenario per iteration

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Theorem

Algorithm terminates after a finite number of iterations with a global optimal solution or proves infeasibility.

How Many Scenarios Do We Need?

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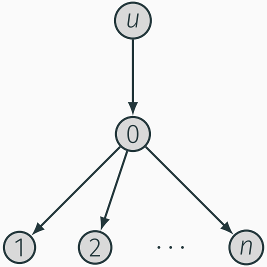

$$\varphi_a(q_a) = \Lambda_a q_a |q_a|, \; \Lambda_a = 1,$$

$$[\pi_w^-, \pi_w^+] = [1, 5]$$

Source u, sinks $1, \ldots, n$, inner node 0

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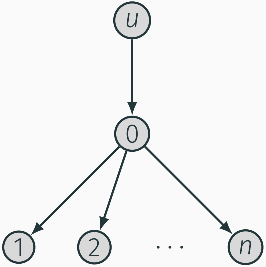

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Parallel expansion candidates

Box uncertainty set

$$U = \{\ell_w \in [0,2], w \in V,\ \ell_0 = 0\} \cap \{\ell_u = \textstyle\sum_{v \in V_-} \ell_v\}$$

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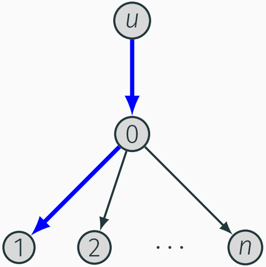

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1. Iteration
    - Worst-Case demand: $d_u = d_1 = 2$ remaining nodes demand 0

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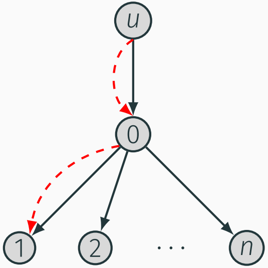

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- Worst-Case demand: $d_u = d_1 = 2$ remaining nodes demand 0
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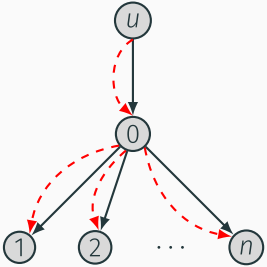

After n-iterations

$|V_+| \times |V_-|$ worst-case scenarios

$$S = \{\ell_u = \ell_v = 2,\ \ell_w = 0, w \in V_- \setminus \{v\} \text{ for all } v \in V_-\}$$

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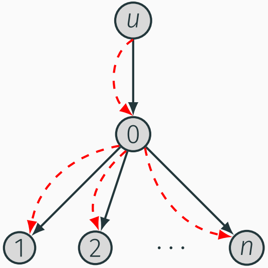

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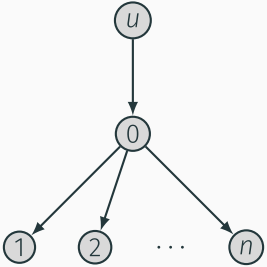
Why do we need “so many” worst-case scenarios?

$\rightarrow$ Limited supply capacity of the source

How Many Scenarios Do We Need?



Same network with larger supply capacity

$$\tilde{U} = \{\ell_v \in [0,2], v \in V_-,\ \ell_0 = 0,\ \ell_u \leq 2|V_-|\} \\ \cap \{\ell_u = \textstyle\sum_{v \in V_-} \ell_v\}$$

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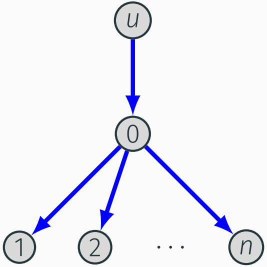

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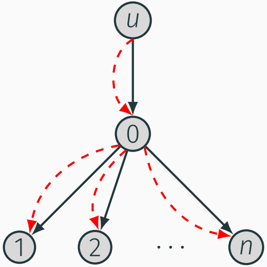

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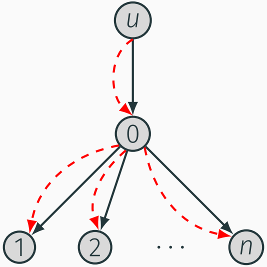
1. Iteration:
   - Worst-case scenario $d_u = 2|V_-|, d_i = 2, i \in \{1, \dots, n\}$
   - Algorithm terminates after a single iteration

How Many Scenarios Do We Need?



Same network with larger supply capacity

$$\tilde{U} = \{\ell_v \in [0,2], v \in V_-,\ \ell_0 = 0,\ \ell_u \leq 2|V_-|\}$$
$$\cap \{\ell_u = \textstyle\sum_{v \in V_-} \ell_v\}$$

1. Iteration:
   - Worst-case scenario $d_u = 2|V_-|$, $d_i = 2, i \in \{1, \dots, n\}$
   - Algorithm terminates after a single iteration

In real-world utility networks sources can supply many sinks
$\rightarrow$ very few worst-case scenarios in practice

Computational Results

Computational Setup

Implemented in Python 3.7 and Pyomo 6.7.0

Solving MINLPs with Gurobi 10.0.3

Time limit of 24 hours per instance

Gas networks $\varphi_a = \Lambda_a q_a |q_a|$

Expansion candidates are in parallel with up to four different diameters

instance	#nodes	#sources	#sinks	#pipes	#short pipes
GasLib-40	40	3	29	39	6
GasLib-60	60	3	39	61	18

Computational Results

Consider four different polyhedral uncertainty sets
$\rightarrow$ with and without correlations between sinks

Add to the plain algorithm

- Acyclic inequalities (Habeck and Pfetsch 2022)
- Mixed-integer convex relaxation $\rightarrow$ lower bounds for the MINLPs

$\rightarrow$ only used for computing lower bounds

General approach is exact

Robustifying Existing Networks

Robustifying Existing Networks

Plain Approach (Left: GasLib-40, Right: GasLib-60)

#Solved	4 of 4		
	Min	Median	Max
#Scenarios	1	2	2
Runtime (s)	807.65	1395.33	1578.68

#Solved	4 of 4		
	Min	Median	Max
#Scenarios	1	1	1
Runtime (s)	1117.37	1175.83	3009.57

Approach with lower bound strengthening

#Solved	4 of 4		
	Min	Median	Max
#Scenarios	1	2	2
Runtime (s)	332.21	1149.98	2042.90

#Solved	4 of 4		
	Min	Median	Max
#Scenarios	1	1	1
Runtime (s)	564.06	995.62	1037.74

Greenfield Approach

Greenfield Approach

Plain Approach (Left: GasLib-40, Right: GasLib-60)

#Solved	1 of 4		
	Min	Median	Max
#Scenarios	1	1	1
Runtime (s)	7320.85	7320.85	7320.85

#Solved	1 of 4		
	Min	Median	Max
#Scenarios	2	2	2
Runtime (s)	81 895.84	81 895.84	81 895.84

Approach with lower bound strengthening

#Solved	3 of 4		
	Min	Median	Max
#Scenarios	1	3	3
Runtime (s)	4066.79	39 963.87	50 183.53

#Solved	1 of 4		
	Min	Median	Max
#Scenarios	2	2	2
Runtime (s)	51 290.35	51 290.35	51 290.35

Summary and Outlook

An algorithm to compute adjustable robust network designs for nonlinear flows

- Finitely many “worst-case scenarios”
- Finite termination for arbitrary compact uncertainty sets
- Approach performs well in practice

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An algorithm to compute adjustable robust network designs for nonlinear flows

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Future research

- Extension to active elements
- Valid inequalities for network design problems with potential-based flows

Main Source

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J. Thürauf, J. Grübel, and M. Schmidt (2024). *Adjustable Robust Nonlinear Network Design under Demand Uncertainties*. Tech. rep. URL: https://optimization-online.org/?p=26035

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