

# Comparing Profit-Maximizing Offer Behavior of Generators in Centrally Versus Self-Committed Wholesale Electricity Markets

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# Introduction, Background, and Research Question

- A dichotomy in electricity-market design: who makes unit-commitment decisions
- U.S. markets have evolved towards **centrally committed** designs—the market operator (MO) collects complex multi-part offers and solves unit-commitment problem to co-ordinate these decisions
- Other markets use **self-committed** designs—generators determine unit commitments individually and MO clears demand against supply based on simple energy offers
- Centralized commitment is more efficient if the auction is incentive-compatible [Sioshansi et al., 2008b]
- **Research Question:** How do the two market designs compare, accounting for incentive properties?

## Self-Committed Design

$$\max \sum_{t \in \mathcal{T}} \left[ (\omega_t - c_i^v) x_{i,t} - c_i^f u_{i,t} \right]$$

$$\text{s.t. } 0 \leq b_j^v \leq \bar{b}^v$$

$$u_{i,t} \in \{0, 1\}; \forall t \in \mathcal{T}$$

(1)-(3)

where:

$$\min \sum_{j \in \mathcal{G}, t \in \mathcal{T}} b_j^v x_{j,t} \quad (1)$$

$$\text{s.t. } \sum_{j \in \mathcal{G}} x_{j,t} = D_t; \forall t \in \mathcal{T} \quad (\omega_t) \quad (2)$$

$$0 \leq x_{j,t} \leq K_j u_{j,t}; \forall j \in \mathcal{G}, t \in \mathcal{T} \quad (3)$$

- Impose some standard assumptions
- Transform bi-level self-committed model into a single-level problem by replacing lower-level market-clearing problem (1)–(3) with its necessary and sufficient KKT conditions

# Centrally Committed Design

$$\begin{aligned} & \max \sum_{t \in \mathcal{T}} \left[ (\eta_t - c_i^v) x_{i,t} - c_i^f u_{i,t} \right] \\ & \text{s.t. } 0 \leq b_i^v \leq \bar{b}^v \\ & \quad 0 \leq b_i^f \leq \bar{b}^f \\ & \quad (4)-(7) \end{aligned}$$

where:

$$\min \sum_{j \in \mathcal{G}, t \in \mathcal{T}} \left( b_j^v x_{j,t} + b_j^f u_{j,t} \right) \quad (4)$$

$$\text{s.t. } \sum_{j \in \mathcal{G}} x_{j,t} = D_t, \forall t \in \mathcal{T} \quad (5)$$

$$0 \leq x_{j,t} \leq K_j u_{j,t}; \forall j \in \mathcal{G}, t \in \mathcal{T} \quad (6)$$

$$u_{j,t} \in \{0, 1\}; \forall j \in \mathcal{G}, t \in \mathcal{T}; \quad (7)$$

- Lower-level market-clearing problem (4)–(7) is mixed-integer, so there are no simple optimality conditions with which to convert this to a single-level problem
- **Added Wrinkle:** Centrally committed designs use make-whole payments:

$$\max \left\{ 0, \sum_{t \in \mathcal{T}} [(b_i^v - \eta_t)x_{i,t} + b_i^f u_{i,t}] \right\}$$

to mitigate reported economic confiscation [O'Neill et al., 2005, Sioshansi, 2014].

# General Approach [Huppmann and Siddiqui, 2018]

- General mixed-binary problem:

$$\begin{aligned} \min & f(x, y) \\ \text{s.t.} & h(x, y) = 0 \\ & g(x, y) \leq 0 \\ & x \in \mathbb{R}^n, y \in \{0, 1\}^m \end{aligned}$$

- If we fix  $y = \bar{y}$ , KKT conditions for  $x$  are as usual:

$$\begin{aligned} \nabla_x f(x, \bar{y}) + \lambda^\top \nabla_x h(x, \bar{y}) + \mu^\top \nabla_x g(x, \bar{y}) &= 0 \\ h(x, \bar{y}) &= 0 \\ g(x, \bar{y}) &\leq 0 \perp \mu \geq 0 \end{aligned}$$

- Solution technique:

- 1 Enumerate all possible  $\bar{y}$ , gives a set  $\mathcal{Y}$
- 2 For each  $y \in \mathcal{Y}$  find associated  $x^*(y)$ ,  $\lambda^*(y)$ ,  $\mu^*(y)$  using KKT conditions
- 3 Select the best  $x^*(y)$  &  $y$

- This gives a single-level mixed-binary (usually nonlinear) problem, with the number of auxiliary variables and KKT conditions growing exponentially with  $m$

# Previous Literature

- Centralized commitment finds near-optimal solutions with different prices and generator profits [Johnson et al., 1997, Sioshansi et al., 2008a, Sioshansi and Tignor, 2012]
- Comparison of the two designs *vis-à-vis* supply and demand flexibility, resource remuneration, and market power and efficiency [Ahlqvist et al., 2022]
- Aforementioned works assume truthful revelation by generators
  - ➔ Limited works that consider strategic offering behavior and incentive properties

# Key Findings

- With symmetric duopoly and single operating period, the offer caps markets can be set so the two designs are expected-cost equivalent [Sioshansi and Nicholson, 2011]
- This equivalence breaks-down with multi-firm oligopoly, due to uniform-price requirement of a self-committed design [Duggan, Jr. and Sioshansi, 2019]
  - ➔ Price under self-committed design must be high enough for the marginal generator to recover its fixed cost, which yields positive economic rents to inframarginal generator(s)
- Higher cost **and productive-efficiency losses** of self-committed design with asymmetric firms
  - **Discriminatory** make-whole payment provides an additional degree of freedom for rent-seeking behavior under centrally committed design
  - Under self-committed design, the only avenue for rent-seeking is to increase the uniform energy price

# Contributions

- Relax partially the symmetry assumption by allowing generators with different costs but same capacities
- Compute partial equilibrium—profit-maximizing offers for one firm, holding rival offers fixed
- Capture multiple operating periods that are linked by long-lived offers
- **Key technical contribution:** an efficient approach to solving profit-maximization for a centrally committed market design



# Solution Approach

## Overview

- Because of symmetric-capacity assumption ( $K_j = K, \forall j \in \mathcal{G}$ ), an optimal solution to the MO's problem results in each generator being either:
  - inframarginal ( $x_{j,t} = K$ ),
  - marginal ( $x_{j,t} = r_t$ ), or
  - inactive ( $x_{j,t} = 0$ )during each  $t \in \mathcal{T}$
- Thus, generator  $i$ 's optimal offers yields one of only  $3^{|\mathcal{T}|}$  candidate production profiles
- For each candidate production profile,  $\hat{x}_i$ , we have a necessary and sufficient constraint set,  $\mathcal{B}_{\hat{x}_i}$ , which characterizes generator- $i$  offers that make  $\hat{x}_i$  optimal in MO's problem
- For each candidate  $\hat{x}_i$ , solve an auxiliary problem with the constraint set,  $\mathcal{B}_{\hat{x}_i}$ , to determine offers that yield  $\hat{x}_i$  as a production profile and resultant maximized profit

# Example

## Non-Zero Rival Fixed Costs

- Three firms, three time periods
- Capacities:  $K = 20$  MW
- $c_j^f = \$10$ ,  $c_j^v$  varies

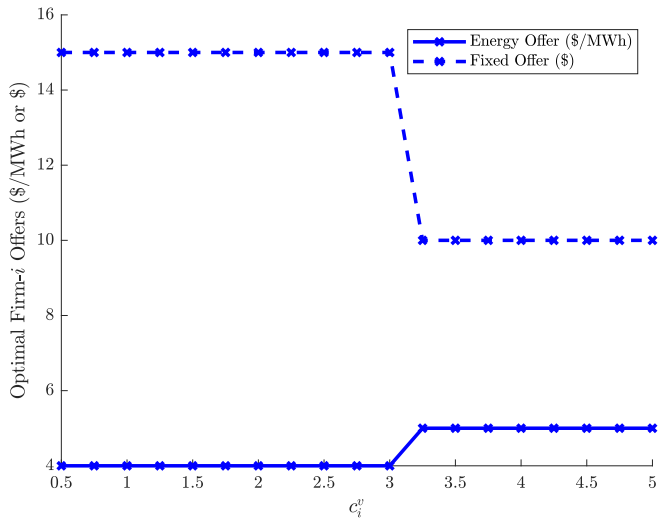
Table: Cost Data

$j$	$c_j^v$	$c_j^f$
1	4	10
2	5	10

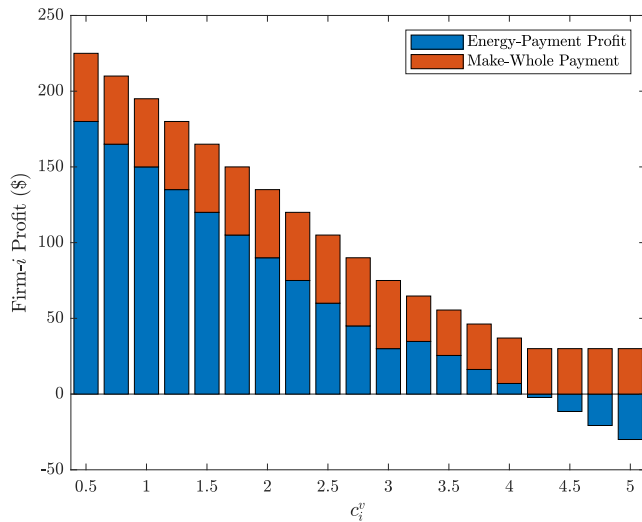
Table: Demand Data

$t$	$D_t$
1	25
2	34
3	38

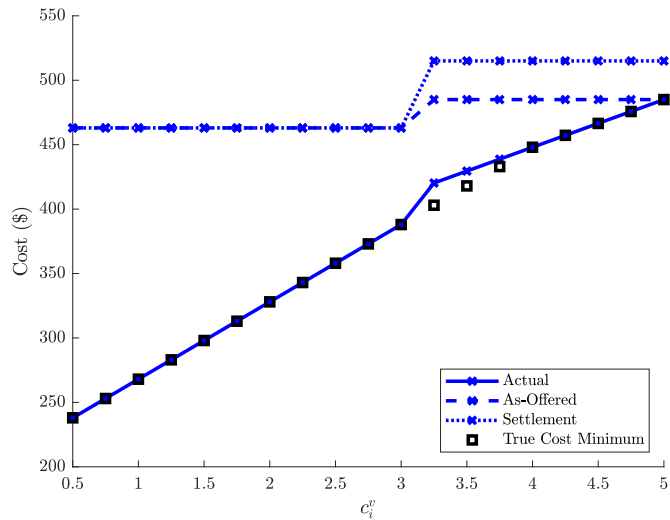
# Optimized Offers



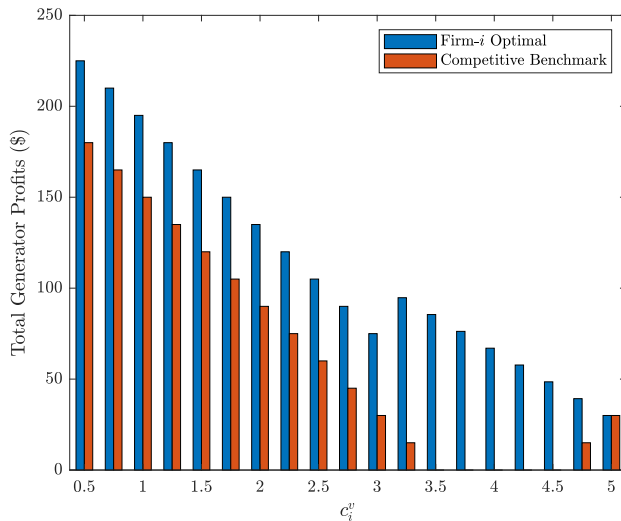
# Firm- $i$ Profit



# Operation Cost



# Total Profit



# Example

## Zero Rival Fixed Costs

- Three firms, three time periods
- Capacities:  $K = 20$  MW
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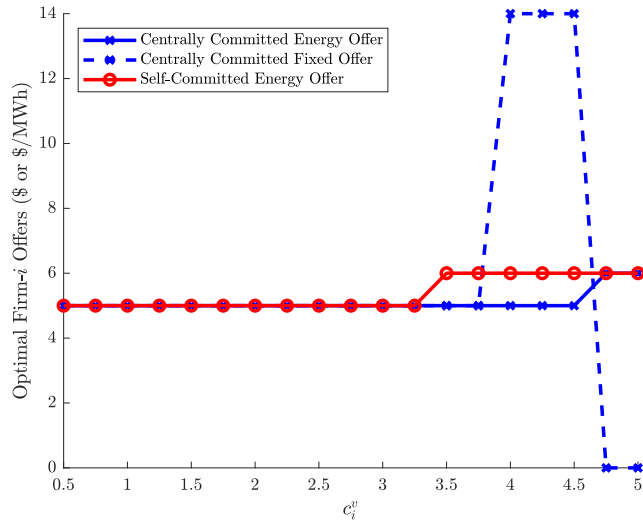
Table: Cost Data

$j$	$c_j^v$	$c_j^f$
1	5	0
2	6	0

Table: Demand Data

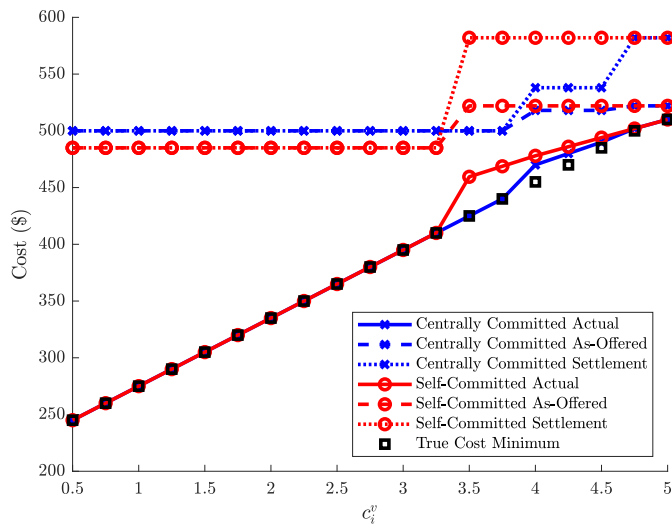
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# Optimized Offers

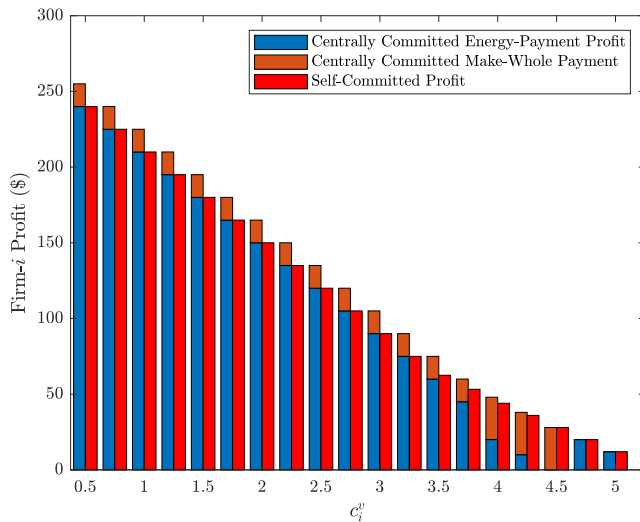




# Operation Cost



# Optimized Profit



## Computational Performance

- Generate random instances of problem with different numbers of firms,  $|\mathcal{G}|$ , and hours,  $|\mathcal{T}|$
- Solve each with 12-hour time limit
- Programmed using Python 3.7 and solved with Gurobi 9.1.1
- Computer with two 2.90-GHz cores and 16.0 GB memory

**Table:** Average Computation Time (s)

$ \mathcal{G} $	$ \mathcal{T} $	[Huppmann and Siddiqui, 2018]	Proposed Algorithm
2	2	0.176	0.066
2	3	2.572	0.098
2	4	70.073	0.212
2	5	5827.641	0.319
3	2	5.167	0.086
3	3	1159.978	0.153
3	4	$\infty$	0.296
3	5	$\infty$	0.660
4	2	$\infty$	0.061
4	3	$\infty$	0.135
4	4	$\infty$	0.275
4	5	$\infty$	0.531
5	2	$\infty$	0.068
5	3	$\infty$	0.146
5	4	$\infty$	0.302
5	5	$\infty$	0.651

# To Summarize and Conclude

- Self-committed designs appear to be more expensive to consumers and have greater productive-efficiency losses
- Firms exercise market power in a self-committed design solely through raising energy prices, which are paid to everyone
- Make-whole payments in centrally committed design give generators a discriminatory mechanism for rent-seeking
- **Some unanswered questions:**
  - How do these comparisons change with multiple profit-maximizing firms (*i.e.*, complete equilibrium)?
  - Absent a complete equilibrium, incorporate uncertainty into firm  $i$ 's problem

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*Thank you!*