Equilibrium Modeling in Natural Gas Markets: A Theoretical Analysis and a Case Study for Brazil

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Outline

U.S. and European Gas Market Liberalization

- United States natural gas market liberalization, regulatory action in 1985 and 1992 by the Federal Energy Regulatory Commision (FERC)
- Full deregulation of the British natural gas market in 1998
- Continental Europe, Directive 98/30/EC (2000), then a second directive (2003/55/EC) to speed up liberalization, for example:
    - Third-Party Access to Essential Infrastructure
    - New Market Players for the gas supply chain

https://inis.iaea.org/collection/NCLCollectionStore/_Public/38/027/38027810.pdf.

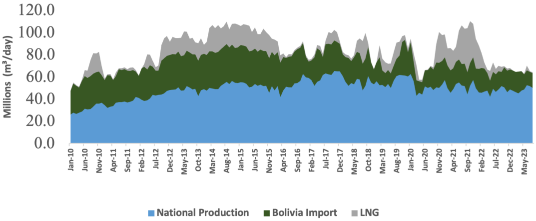
The Brazilian Gas Market In Transition, Recent Regulatory Initiatives

- **In 2016**, the federal government launched the "Gás para Crescer" (Gas to Grow) initiative, which outlined strategies for a new natural gas market with the goal of creating a competitive environment in the sector.
- **In 2019**, the federal government introduced the "Novo Mercado de Gás" (New Gas Market - NMG) program, developed in collaboration with various Brazilian government institutions,
  - to facilitate the integration of the gas sector with the electricity sector
  - to harmonize state and federal regulations
  - to remove tax barriers, and enhance the regulation of gas transportation and distribution to ensure competitiveness and competition in the sector

IEA (2021), Carvalho et al. (2021), Fernandes (2022), Guitierrez (2022), Silva et al. (2023).

Brazilian Gas Supply in 2022

- 138 million m^3/d =gross volume, 47.6 million m^3/d= effective amount available for the market (after reinjection of associated gas)
  - domestic production (both offshore-84% and onshore) mostly associated gas (87%)
  - imports from Bolivia via pipelines (2022 average of 17.5 million m^3/d)
  - purchases from the international LNG market through five operational LNG terminals
- In November 2023,
  - Petrobras produced 66.4% of Brazil's natural gas
  - Other producers include: the domestic company Eneva (3.1%) and four major international companies: Shell (11.3%), Total (3.9%), Galp (3.1%), and Repsol (1.3%)

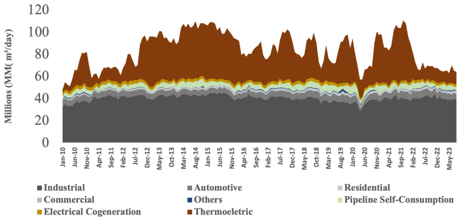


Brazilian Gas Supply

MME (2023), ANP (2023).

Brazilian Gas Demand

- Regulated distributors, delivery through the distribution system to end consumers including:
  - industrial, automotive, commercial, and electricity generation sectors (with power plants using or not using the distribution system)
- Non-regulated (free) consumers (small %)



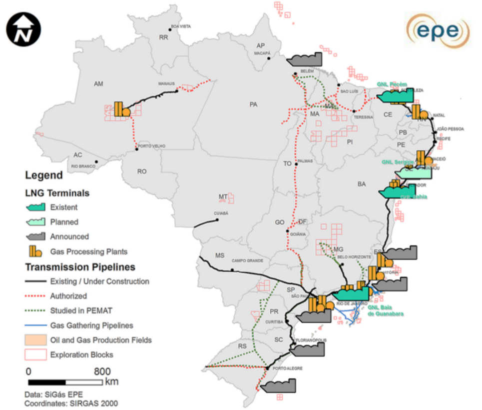
Brazilian Gas Demand

MME (2023), ANP (2023).

Brazilian Gas Infrastructure

- The Brazilian gas market includes:
  1. producers, 2. LNG importers, 3. gas from Bolivia, 4. consumers.
- We will concentrate on producers and consumers for this presentation.
- Each producer solves an appropriate optimization problem (e.g., profit maximization).
- Consumers are modeled via inelastic or elastic consumption.
- Also market-clearing conditions.

Figure 5 • **Brazil's gas infrastructure**



This map is without prejudice to the status of or sovereignty over any territory, to the delimitation of international frontiers and boundaries and to the name of any territory, city or area.

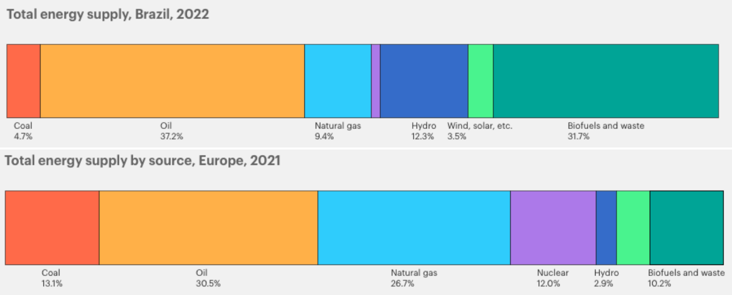
Source: EPE (2018), "Presentation to the IEA on natural gas"

EPE (2018)

NU
Norwegian University of Science and Technology

Contextualizing Brazil's Energy Supply

- Europe is the third largest supplier of energy in the world, accounting for 13% of the world's energy supply. Comparatively, Brazil is the 7th largest supplier, supplying 2.0% of the world's energy.[1]
- Natural gas comprises 9.4% of Brazil's energy mix, while it makes up 26.7% of EU energy supply.

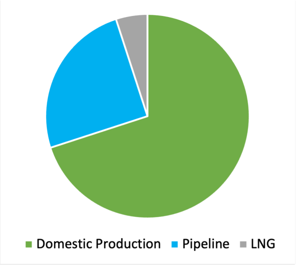


[1] Total energy supply includes all energy produced in or imported to a country, minus that which is exported or stored.

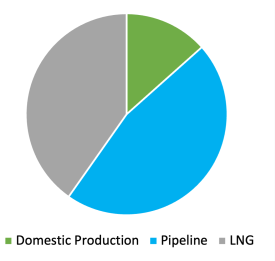
IEA (2023)

Brazil & Europe Natural Gas Supply

- In 2022, Brazil NG production was 138 million m^3/day. Domestic NG production represents 70% of this supply, while pipeline imports from Bolivia contribute 25% followed by LNG imports at 5%.
- In 2023, EU available supply was 898 million m^3/day, with 13% coming from domestic production, 46% from pipeline imports from Norway, North Africa, UK and Azerbaijan and 40% from LNG imports.



Brazil Gas Supply

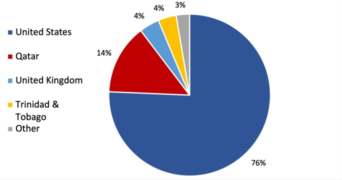


EU Gas Supply

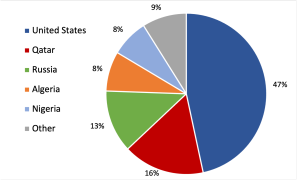
ITA (2023), IEA (2023)

Brazil & Europe LNG Imports

- Between 2016-2022, Latin America received 15.7% of U.S. LNG exports, while exports to Europe and Central Asia comprised 42.6% of cumulative U.S. LNG exports.
- Brazil is the 9th largest U.S. LNG importer, receiving more U.S.-produced LNG than all but three EU nations between 2016-2022 (Spain, France and the Netherlands).
- The U.S. is the largest LNG exporter to both Brazil and the EU, accounting for 76% of Brazil's total LNG imports and 47% in the EU during 2022.



Brazil LNG Imports



EU LNG Imports

Source: DOE (2023), ITA (2023), Brazil Ministry of Economy Database, IEEFA

Nash Equilibrium, Price-Makers and Price-Takers

- If each producer takes into account the other players' optimal values $\rightarrow$ Nash equilibrium.
- This is usually done via an inverse demand function of the form $\pi_t(x) = \alpha_t - \beta_t \left(\sum_p x_t^p\right)$ or price-maker formulation where:
  - $\alpha_t, \beta_t > 0$ are known constants of the inverse demand function at time $t \in T = \{1, \ldots, n_t\}$ (it may be stationary over time)
  - x_t^p is the production decision of producer $p \in P = \{1, \ldots, n_p\}$ in time $t \in T$ with $x = (x^p : p \in P)$, $x^p = (x_1^p, \ldots, x_{n_t}^p)^T$
  - $\pi_t(x)$ is the resulting market price allowing for market power by the producers to maximize their profit $\sum_t(\pi_t(x) \cdot x_t^p)$
- Otherwise, the producer is a price-taker and the market price π_t is data to each producer and the profit is given by $\sum_t(\pi_t \cdot x_t^p)$.
- In the price-taker formulation, the price π is determined by market-clearing conditions in this case.

Gabriel et al. (2012), Gabriel et al. (2024)

Linear Complementarity Problems

- The linear complementarity problem (LCP) is defined as:

$$F(z) = Mz + q \quad (1)$$

for a matrix M and vector q

- In that case, the LCP can be succinctly written as follows: find $z \in \mathbb{R}^s$ so that

$$0 \leq Mz + q \perp z \geq 0 \quad (2)$$

Note: In the following definitions, the matrix A doesn't need to be symmetric.

Definition

Let A be a square matrix of size $n \times n$. Then A is a positive semi-definite (PSD) matrix if $z^T Az \geq 0 \ \forall z \in \mathbb{R}^n$.

Definition

Let A be a square matrix of size $n \times n$. Then A is a positive definite (PD) matrix if $z^T Az > 0 \ \forall z \neq 0, \in \mathbb{R}^n$.

Cottle et al. (2009).

Theorems

Here are the relevant theorems from [Cottle et al., 2009] summarized in one theorem below.

Theorem

Consider the LCP (2). Then, the following results hold:

- *(Theorem 3.1.6) If M is positive definite, then there is a unique solution for all right-hand sides $b \in \mathbb{R}^n$.*
- *(Theorem 3.1.2) If M is a positive semi-definite matrix, then if the LCP is feasible then it is solvable. (Feasibility for the LCP means: $\exists z \geq 0$ such that $q + Mz \geq 0$).*
- *(Theorem 3.1.7. part c) If M is positive semi-definite and has a solution $\bar{z}$, then the set of all solutions is given by the following polyhedral constraints $S^{solutions}(\bar{z})$:*

$$S^{solutions}(\bar{z}) = \{z \in \mathbb{R}^n : z \geq 0, Mz+q \geq 0, q^T(z-\bar{z}) = 0, (M+M^T)(z-\bar{z}) = 0\} \quad (3)$$

PSD Case, Finding Different Equilibria

- Having a strictly convex function $c(z)$ and a given LCP solution $\bar{z}$, we can solve the strictly convex program:

$$\min_{z} \quad c(z) \tag{4a}$$
$$\texttt{s.t.} \quad (M + M^T)(z - \bar{z}) = 0 \tag{4b}$$
$$Mz + q \geq 0, \tag{4c}$$
$$q^T(z - \bar{z}) = 0, \tag{4d}$$
$$z \geq 0 \tag{4e}$$

- (4) will always have a unique solution z^* for a given vector strictly convex function $c(z)$, $S^{solutions}(\bar{z})$ is the feasible region to this problem
- If the solution to this problem is z^* and it equals $\bar{z}$, then relative to the metric $c(z)$ it is a unique solution.
- Two examples of $c(z)$ are: social welfare maximization and carbon-dioxide emissions minimization.

Gabriel et al. (2024).

PSD Case, Finding Different Equilibria Maximizing Social Welfare

- The social welfare of an equilibrium solution is given by the integral of the inverse demand function, minus the costs of production, summed over all firms:

$$SW(z) = \alpha \sum_{p \in P} x^p - \frac{1}{2}\beta \left(\sum_{p \in P} x^p \right)^2 - \sum_{p \in P} \gamma^p x^p, \tag{5}$$

  where x^p is the vector production levels for producer p for times $1, \ldots, n_t$, $\alpha, \beta > 0$ are the slope and intercept of the linear inverse demand curve, respectfully, and γ^p is a vector of production costs for producer p in each time period.

- The expression for social welfare is strictly concave; thus, to maximize social welfare we set $c(z) = -SW(z)$, which is strictly convex.

Ruiz et al. (2014), Gabriel et al. (2024).

PSD Case, Finding Different Equilibria Minimizing Carbon-Dioxide Emissions

- Suppose that the emissions of gas producer p are given by a quadratic emissions function

$$EM(z) = \sum_{p \in P} (x^p)^\top \Lambda^p x^p + (\eta^p)^\top x^p + \delta^p, \quad (6)$$

where Λ^p for player p is a diagonal matrix with nonnegative entries, η^p is a vector of emissions coefficients and δ^p is a scalar.

- Thus, setting $c(z) = EM(z)$ gives a strictly convex function that can be used to determine a unique equilibrium solution that minimizes total emissions.

Soroudi (2017), Gabriel et al. (2024).

2-Variable Example

- Consider the following 2-variable LCP:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \leq \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} -9 \\ -8 \end{pmatrix} \perp \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (7)$$

  Since the matrix M is positive definite, there exists exactly 1 solution to this LCP.
  - The unique solution is $z^* = (2/3, 7/15)^\top \approx (0.667, 0.467)^\top$.
- As a result, the optimal solution to QP (4) will be z^* for any feasible choice of c:

c	$(1,1)$	$(1,2)$	$(2,1)$	$(0,1)$	$(1,0)$
z^*	$(2/3, 7/15)$	$(2/3, 7/15)$	$(2/3, 7/15)$	$(2/3, 7/15)$	$(2/3, 7/15)$

Allen (2022).

2-Variable Example *cont.*

- Now, consider the following 2-variable LCP with *infinitely many* solutions:

$$\begin{pmatrix}0\\0\end{pmatrix} \leq \begin{pmatrix}1 & 2\\2 & 4\end{pmatrix}\begin{pmatrix}z_1\\z_2\end{pmatrix} + \begin{pmatrix}-4\\-8\end{pmatrix} \perp \begin{pmatrix}z_1\\z_2\end{pmatrix} \geq \begin{pmatrix}0\\0\end{pmatrix} \tag{8}$$

- Any convex combination of $(4,0)$ and $(0,2)$ are solutions to this LCP:

$$z^* = \lambda_1 \begin{pmatrix}4\\0\end{pmatrix} + \lambda_2 \begin{pmatrix}0\\2\end{pmatrix}, \tag{9}$$

such that $\lambda_i \geq 0, i = 1,2$ and $\lambda_1 + \lambda_2 = 1$. Hence, given a solution $\bar{z}$ to LCP (8), varying the vector c in QP (4) yields different solutions z^*:

c	$(1,1)$	$(1,2)$	$(2,1)$	$(0,1)$	$(1,0)$
z^*	$(4/5, 8/5)$	$(4/3, 4/3)$	$(4/9, 16/9)$	$(4,0)$	$(0,2)$
λ	$(1/5, 4/5)$	$(1/3, 2/3)$	$(1/9, 8/9)$	$(1,0)$	$(0,1)$

Allen (2022).

Gas Market Structure

We consider a set of natural gas firms (players) $P = \ldots 1, \ldots, n_p$ and the following diametrically opposed situations:

1. An LCP market equilibrium problem for which each player is a price-taker and maximizes profit and prices are formed at a hub over a set of times periods $T = \{1, \ldots, n_t\}$.
2. An LCP market equilibrium problem for which each player is a price-maker and maximizes profit and prices are formed for times periods $T = \{1, \ldots, n_t\}$ via a linear, inverse demand function expressing market power so that the players can manipulate prices via their supply levels.
3. These n_p firms constitute the entire natural gas market for a given region and can be production, LNG, storage, or other players in this market.
4. Can also have elastic or inelastic demand (or mixtures) as well as convex combinations of price-taking and price-making suppliers (e.g., market power mitigation aspects)
5. Thus, several cases to consider.

Gabriel et al. (2024).

Price-Taker Optimization Problem with Inelastic Demand

Each player (firm) $p \in \{1, \ldots, n_p\}$ maximizes profit by solving a quadratic program with linear constraints of the following form:

$$min_{x^p} f_p(x^p) = \left(\frac{1}{2}(x^p)^T Q^p x^p + (c^p)^T x^p + k^p\right) - \pi^T x^p \quad (10a)$$

$$\texttt{s.t.} \quad -A^p x^p + b^p \leq 0 \quad (\lambda^p) \quad (10b)$$

$$-x^p \leq 0 \quad (10c)$$

where

- x^p is the set of variables (e.g., production) under control of player p for the time periods $t \in T = \{1, \ldots, n_t\}$, i.e., $x^p = (x_1^p, \ldots, x_{n_t}^p)$
- Q^p, c^p, k^p are respectively, the quadratic part of player p's cost function (i.e., the Hessian matrix), the linear portion, and the constant term so that $\left(\frac{1}{2}(x^p)^T Q^p x^p + (c^p)^T x^p + k^p\right)$ measures the total cost
- $\pi = (\pi_1, \ldots, \pi_{n_t})$ is the vector of prices for each time period involved to be determined by the virtual hub and not variables for player p but variables in the overall LCP (see below for details)
- the matrix A^p and the vector b^p are the data in the constraints of the problem and without loss of generality can include equations (expressed as two inequalities)
- λ^p are the multipliers to their associated linear constraints

Gabriel et al. (2024).

Market-Clearing Conditions

- Market-clearing at the hub is of the form:

$$0 \leq \sum_p x_t^p - D_t \perp \pi_t \geq 0, t = 1, 2 \qquad (11)$$

- where D_t is the total, exogenous data that is **inelastic**.

Gabriel et al. (2024).

Example with Two Players & Two Time Periods

Consider the following example of with two players $P = \{1, 2\}$ and two time periods $T = \{1, 2\}$. We let

$$Q^1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, Q^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

which are both positive semi-definite and let

$$c^1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, c^2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

We also include for each player $p = \{1, 2\}$, 3 constraints as follows:

$$\sum_t x_t^p \leq x^{p,total} \quad \texttt{volumetric constraint} \tag{12a}$$

$$x_t^p \leq x_t^{p,rate}, t = 1, 2 \quad \texttt{production rate constraint} \tag{12b}$$

Gabriel et al. (2024).

Resulting LCP for 2 Players and 2 Periods → 12 Variables

Thus, the vector $z^1 \in \mathbb{R}^{12}$ and is given as:

$$z = \left(x_1^1, x_2^1, \lambda_1^1, \lambda_2^1, \lambda_3^1, | x_1^2, x_2^2, \lambda_1^2, \lambda_2^2, \lambda_3^2, | \pi_1, \pi_2\right)^T$$

In this case, the LCP matrix M is psd and is 12×12 and given as:

$$M = \left(\begin{array}{ccccc|ccccc|cc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right), q = \begin{pmatrix} 0 \\ 0 \\ x^{1,total} \\ x_1^{1,rate} \\ x_2^{1,rate} \\ 0 \\ 0 \\ x^{2,total} \\ x_1^{2,rate} \\ x_2^{2,rate} \\ -D_1 \\ -D_2 \end{pmatrix}$$

Gabriel et al. (2024).

Elastic Demand

Now we consider the case of an **elastic inverse demand function** assuming $\alpha_t, \beta_t > 0, \forall t$

$$\pi_t = \alpha_t - \beta_t \left(\sum_p x_t^p \right) \quad (13)$$

and the resulting, adjusted market-clearing conditions

$$0 \leq \sum_p x_t^p - \left(\frac{\alpha_t - \pi_t}{\beta_t} \right) \perp \pi_t \geq 0 \quad (14)$$

Note: The new demand is **elastic** and is given as

$$D_t(\pi_t) = \left(\frac{\alpha_t - \pi_t}{\beta_t} \right) \quad (15)$$

Gabriel et al. (2024).

Theoretical Results Pricer-Makers & Inelastic or Elastic Demand

- **Price-taker, elastic demand**: We can replace π_t from (11) with π_t from (14)
- **Price-maker**: We can replace π_t as data for each producer and instead use (13) in the objective function (10a), this will be a convex QP if $\beta_t > 0$
- **Convex combination** of price-taker (inelastic or elastic) and price-maker:
  - Each producer $p \in P$ solves the following convex QP:

$$\min_{x^p} \quad \theta f^{p,PT}(x^p) + (1-\theta) f^{p,PM}(x^p) \tag{16a}$$

$$\text{s.t.} \quad A^p x^p \geq b^p \qquad (\lambda^p) \tag{16b}$$

$$x^p \geq 0, \tag{16c}$$

where $f^{p,PT}$ and $f^{p,PM}$ are producer p's quadratic cost functions from the price-taker and price-maker models, respectively.

Gabriel et al. (2024).

Example: Price-Maker LCP Matrix Structure

Let M^{PM} be the LCP matrix coming from the KKT conditions for all price-making producers. Then, for $N^{PM} = M^{PM} + (M^{PM})^\top$

$$N^{PM} = \begin{bmatrix} \begin{pmatrix} 4D & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} & \cdots & \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 4D & 0 \\ 0 & 0 \end{pmatrix} & \cdots & \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} \\ \ddots & \ddots & \ddots & \\ \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 2D & 0 \\ 0 & 0 \end{pmatrix} & \cdots & \begin{pmatrix} 4D & 0 \\ 0 & 0 \end{pmatrix} \end{bmatrix} \tag{17}$$

and will be of size $(n_t + n_r) * n_p$ where $D = \text{diag}(\beta_1, \ldots, \beta_{n_t})$, where the inverse demand function is $\pi_t = \alpha_t - \beta_t \left(\sum_p x_t^p\right), n_t, n_r, n_p$, are respectively, the number of time periods, constraints in each producer's problem, and number of producers.

Eigenvalues for Price-Taker Matrix N^{PM}

We can show that the eigenvalues of $N^{PM} = M^{PM} + (M^{PM})^\top$ are as follows:

- $\lambda = 0$ is an eigenvalue of $M^{PM} + (M^{PM})^\top$ of multiplicity $n_p * n_r$
- $\lambda = 2\beta_1, 2\beta_2, \ldots, 2\beta_{n_t}$ each is an eigenvalue of multiplicity $n_p - 1$
- $\lambda = 2(n_p + 1)\beta_t$ is an eigenvalue of multiplicity 1 for each $t = 1, \ldots, n_t$

So, since $\beta > 0$, we see that N^{PM} is positive semi-definite.

Theoretical Results

Theorem

Consider the LCP formed from the necessary and sufficient KKT conditions to each producer problem in the case of

- *price-taker with inelastic or elastic demand*
- *price-maker with elastic demand*
- *convex combinations of price-taker and price-maker in the producer objective functions*

Then, the LCP matrix M is positive semi-definite.

Gabriel et al. (2024).

Uniqueness Results

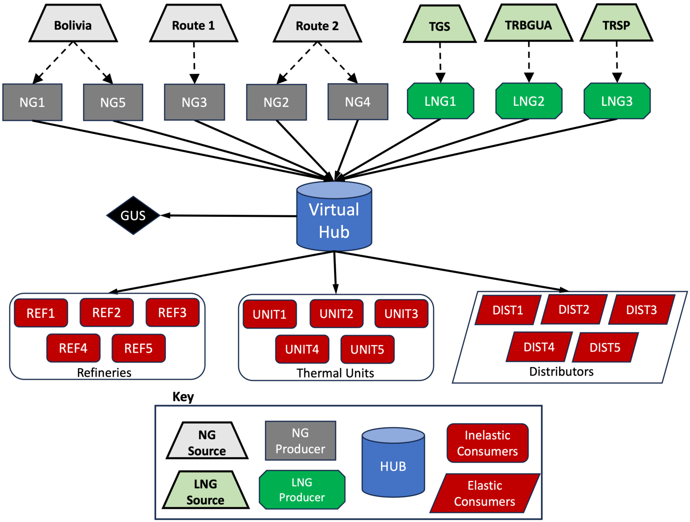
Theorem

Consider the LCP formed from the KKT conditions of the convex combination of the price-taker and the price-maker problems. That is, for producer p and any $\theta \in [0,1]$ an objective function of $\left(\theta f^{p,PM}(x^p) + (1-\theta) f^{p,PT}\right)$ and polyhedral constraints. Then, the following statements hold:

1. *In the* **price-taker case**, *i.e., $\theta = 0$, with either inelastic or elastic demand, the equilibrium production values x^p are unique if Q^p is positive definite.*
2. *In the* **price-taker, elastic demand case**, *then the equilibrium prices π_t are unique.*
3. *In the* **price-maker case**, *i.e., $\theta = 1$, the equilibrium production values x^p are unique and the equilibrium prices given by $\pi_t = \alpha_t - \beta_t(\sum_{p \in P} x_t^p)$ are thus unique as well.*
4. *In the* **convex combination case** *i.e., $\theta \in (0,1)$, for the price-takers with elastic demand, the equilibrium production values and prices are unique if each Q^p is positive semi-definite.*

Gabriel et al. (2024).

Stylized Network



Price-Taker and Price-Maker Results

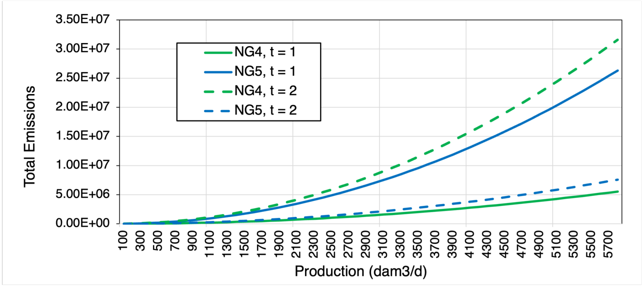
2*Producer	2*Prod. Cost	2*Prod. Cap	Price-Taker ($\theta = 0$)		Price-Maker ($\theta = 1$)	
			x_1^p	x_2^p	x_1^p	x_2^p
LNG1	8.10	14,000.00	0.00	0.00	0.00	0.00
LNG2	9.60	15,000.00	0.00	0.00	0.00	0.00
LNG3	9.10	20,000.00	0.00	0.00	0.00	0.00
NG1	3.00	9,645.00	9,645.00	9,645.00	9,459.88	8,626.07
NG2	3.00	9,555.00	9,555.00	9,555.00	9459.88	8626.07
NG3	3.00	13,100.00	13,100.00	13,100.00	9459.88	8626.07
NG4	3.50	5,145.00	5,145.00	5,145.00	5,145.00	5,145.00
NG5	3.50	9,645.00	9,645.00	7,448.84	8,283.02	7,449.22
Total			47,090.00	44,893.84	41,807.67	38,472.44

Table: Production values for producers in price-taker ($\theta = 0$) and price-maker ($\theta = 1$) formulations for maximum demand scenario, where production quantities and costs are given in cubic decameters per day (dam^3/d) and USD per one million British thermal units (USD/MMbtu), respectively.

2*Scenario	Price-Taker ($\theta = 0$)		Price-Maker ($\theta = 1$)	
	π_1^*	π_2^*	π_1^*	π_2^*
Min. Demand	4.44	4.22	7.40	7.20
Max. Demand	5.28	4.40	7.96	7.60

Table: Market price π^* (USD/MMBtu) in price-taker ($\theta = 0$) and price-maker ($\theta = 1$) formulations.

Example: Minimizing Carbon-Dioxide Emissions



2*Producer	2*Prod. Cost	2*Prod. Cap	Initial LCP Solution		Unique QP Solution	
			x_1^p	x_2^p	x_1^p	x_2^p
NG4	3.50	5,145.00	5,145.00	5,145.00	5,145.00	2,948.84
NG5	3.50	9,645.00	9,645.00	7,448.83	9,645.00	9,645.00
Total			14,790.00	12,593.84	14,790.00	12,593.84

Table: Unique emission minimizing solution for the price-taker model in the maximum demand scenario. Production costs are given in cubic decameters per day (dam^3/d) and costs in US per one million British thermal units (USD/MMTbtu).

Summary

- We considered both price-taker and price-maker formulations for gas producers
- Putting together the KKT conditions of all the producers and market-clearing conditions gives rise to a linear complementarity problem (LCP).
- When the LCP matrix M is positive semi-definite, one can use these results to check for uniqueness relative to a certain metric (via a strictly convex objective function).

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